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Theoretical modeling of attenuated displacement amplification for multistage compliant mechanism and its application

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ABSTRACT

With the merits of multiplying displacement amplification ratio and compact structure, flexure-based multistage compliant mechanisms have been widely proposed in recent ten years. Experimental output displacement, however, is attenuated more or less in various designs and is even reduced to less than 10% of the original ambitious design in some prototypes. In this paper, the issue on attenuated displacement amplification of multistage compliant mechanisms is theoretically investigated. A formula of displacement amplification ratio is established based on the elastic beam theory and by defining an impedance factor, which describes the hindering effect of the second layer on the preceding layer. The high accuracy of the model is verified by finite element analysis with no more than 5% deviations. It allows a designer to quickly get an intuitional sense of why the output displacement is attenuated and how each parameter affects the mechanisms' performance. Thanks to the theoretical guidance, a new planar two-stage compliant mechanism with relatively high frequency response and large range as well as no assembly error is designed and tested. Experiments show a natural frequency of 3.3 kHz in the output direction and displacement of 198 μm, which agrees with the theoretical prediction.

1. Introduction

Piezoelectric-actuated, flexure hinge-based compliant mechanisms are frequently utilized in ultra-precision positioning stages [1] for biological cell manipulation, optical fiber alignment and scanning probe microscope. They are also increasingly applied in many other aerospace and instrumental fields such as shape active control for aircraft [2], fluid control for servo valve [3], fatigue test for microstructures [4], and so forth. These significant applications are attributed to the combination of high resolution, large output force, fast dynamic response of piezoelectric actuators and displacement amplification function of compliant mechanisms without wear, backlash and friction.

Compliant mechanisms, which magnify and/or guide the limited output displacement of piezoelectric actuators, are usually monolithically fabricated by electrical discharge machining process. With the increasing demands on precision actuating with concurrent large range and fast dynamic response (e.g. video-rate scanning probe microscope in real time [5]), great challenges have been posed for design and control of high-bandwidth and large-range compliant mechanisms. Therefore, based on lever-type, rhombus-type, bridge-type and other flexures, many derivative compliant mechanisms such as nested cellular [6], three-dimensional [7], compound bridge-type [8], honeycomb-like [9] compliant mechanisms with high performance and plentiful characteristics were developed in the last decades.

In order to employ the merits of multiplying displacement amplification ratio and compact structure, multistage compliant mechanisms have also been investigated by many researchers. Kim et al. [7,10] proposed a compact three-dimensional two-stage compliant mechanism with high natural frequency. The experimental output displacement, however, was reduced to less than 10% of the original design. Ueda group [6,11,12] designed a piezo-actuated nested multiplying compliant mechanism with an exponential displacement amplification ratio. The experimental output displacement reached to more than 2 mm with the sacrifice of output stiffness and very low natural frequency. Secord and Asada [13] presented a new variable stiffness actuator with tunable resonant frequency by adopting the concept of multistage compliant mechanism. Malosio et al. [14] developed a multistage amplifier based on a parallel mechanism with high natural frequency and high positioning precision. Recently, a two-step displacement ampli-

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fying mechanism was proposed by Guo et al. [15] to develop a two-dimensional micro-vibration table for assisting finishing of micro-optic mold. Also, a piezo-actuated fast *XY* positioner with decoupled motions based on multistage compliant mechanism was designed by Zhu et al. [16] for micro-/nanomachining. Actually, to achieve large workspace and multi-degree of freedom, various multistage compliant mechanisms are increasingly designed for precision positioning stages [17–20].

Output displacements in all aforementioned multistage compliant mechanisms, however, were attenuated more or less. Some works have been conducted to model the static and dynamic behaviors of these multistage compliant mechanisms. For example, the matrix method was used to build the dynamic model of a three-dimensional multistage compliant mechanism [10]; Castigliano's second theorem and a two-port network concept was used to model the input-output static displacement and force relations of a nested compliant mechanism [11]. It should be pointed out that these studies were primarily interested in the input-output behavior of the multistage compliant mechanisms but little concerned about the internal attenuated displacement amplification issues.

For better design of a multistage compliant mechanism, the attenuation of output displacement is theoretically explained (Section 2). Then, a specific analytical formula of attenuated displacement amplification ratio for rhombus-type multistage compliant mechanism is established (Section 3) and verified by finite element analysis (Section 4). Considering the fact that finite element method (FEM) will become burdensome and even powerless for multistage compliant mechanisms with large number of stages, the proposed model provides an intuitional tool to explain why the output displacement is attenuated and how each parameter affects its performance. It is also efficient and convenient for implementing parameter optimization. Based on the theoretical guidance of the proposed model, a new planar two-stage compliant mechanism with no assembly error, relatively high natural frequency and large range is designed, fabricated and tested at last (Section 5). Interestingly, the configuration can be directly utilized as a compact *XY* precision positioning stage with relatively high frequency response and large workspace.

For the scope of this paper, the specific modeling is limited to rhombus-type multistage compliant mechanism for its high natural frequency and large output stiffness. The approach can also be easily generalized to other types of multistage compliant mechanisms with similar conclusions.

2. Theoretical explanation on attenuated displacement

The common essence of different designs of multistage compliant mechanisms is that input port of the next layer is rigidly connected to the output port of the preceding layer. Therefore, input displacement of the next layer is equal to the output displacement of the preceding layer, and the output displacement of the whole mechanism is amplified multiplicatively starting from the first layer to the last layer. According to the size diversity of each layer in multistage compliant mechanisms, it will exhibit extremely large output displacement with larger and thinner outer layer like the nested cellular displacement amplifier proposed by Ueda et al. [6]; as well, output stiffness and natural frequency will be enhanced with a more compact outer layer like the three-dimensional two-stage compliant mechanism proposed by Kim et al. [7].

The schematic diagram and geometric parameters of a typical two-stage compliant mechanism is shown in Fig. 1. This multistage compliant mechanism consists of two monolithic planar rhombus-type flexures, which are nested vertically with each other in the three-dimensional space symmetrically.

As shown in Fig. 1, once driven by an input force, f_{PZT} from the internal piezoelectric actuator in the *x*-direction, the first layer of the two-stage compliant mechanism will produce an output displacement in the *y*-direction under the hindering force, f_{hinder} of the second layer. Again, f_{hinder} is the actuating force of the second layer based on the Newton's third law of motion. Basically, any multistage compliant mechanism is an one-way coupled system in which the output of the former layer is hindered by the next layer but the displacement amplification ratio of the next layer is independent of the former one. And with a large input stiffness of the second layer, the output displacement of the preceding layer will be attenuated heavily. In conclusion, the intrinsic relationship between two layers of arbitrary multistage compliant mechanisms can be summarized as follows:

- The output displacement of the preceding layer under the impedance of the second layer is the input displacement of the second layer.
- The output force of the preceding layer is the actuating force of the second layer.

3. Analytical modeling

As shown in Fig. 1, since the symmetry of the two-stage compliant mechanism, only a quarter of the structure is needed to establish its mechanical model. According to the elastic beam theory and the kinematic characteristics of the multistage compliant mechanism shown in Fig. 1, the static models of the two layers can be modeled in Fig. 2. Here, only flexure arms are considered as flexible and regarded as elastic beams, while other parts of the multistage compliant mechanism are supposed to be rigid body.

Assign f_x and f_y to be the input force of a quarter of the first and second layer, Δx to be the input displacement of the first layer, Δy and Δz to be the output displacement of the first and second layer, respectively. Then, considering the moment equilibrium in the first and second layer, the following relations can be respectively obtained for the two layers:

$$2M_1 = f_x \times L_1 \sin\theta_1 - f_y \times L_1 \cos\theta_1 \tag{1}$$

$$2M_2 = f_y \times L_2 \sin\theta_2 \tag{2}$$

where M_1 and M_2 are moments supplemented at the flexure arm ends to ensure that deflection angles at points A, B and C remain to be zero, respectively. $L_1, \theta_1, L_2, \theta_2$ are structural parameters of the multistage compliant mechanism as shown in Fig. 1.

For the first layer, assign *X* axis is aligned along the neutral axis of the flexure arm AB, then corresponding moment in the elastic beam at the end point *x* is:

$$M_1(x) = M_1 - \left(f_x \times \sin\theta_1 - f_y \times \cos\theta_1\right) \times x \tag{3}$$

Assuming flexure arm AB as an Euler-Bernoulli beam, then work done by input force, f_x exerted by piezoelectric actuator is transformed into three parts according to the principle of conservation of energy: the bending deformation energy and the tensile deformation energy of flexure arm AB, as well as the work done against the impedance of the second layer on the first layer. Therefore, the following Eq. (4) can be obtained for the first layer:

$$\frac{1}{2} f_x \times \Delta x - \frac{1}{2} f_y \times \Delta y = \int_0^{L_1} \frac{f_1^2(x)}{2EA_1} dx + \int_0^{L_1} \frac{M_1^2(x)}{2EI_1} dx \tag{4}$$

where A_1 and I_1 are area and moment of inertia of the corresponding cross-section about the neutral axis of flexure arm AB; *E* is Young's modulus; $f_1(x)$ is axial tension of flexure arm AB in the first layer, which is equal to $(f_x \cos\theta_1 + f_y \sin\theta_2)$ based on the force resolution in Fig. 2.