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Index Reduction and Regularisation Methods for Multibody Systems

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Abstract: This paper presents regularisation methods for differential–algebraic equations of mechanical systems. These systems can be described via systems of differential and algebraic equations that usually have differential index three. Such systems are typically gained by the use of multibody simulation tools, where the system is composed of connected rigid and flexible bodies and different joints and the underlying equations are derived automatically by the software. The considered methods in this paper are applicable to differential–algebraic equations of index three and are divided into three basic approaches: index reduction with differentiation, stabilisation by projection and methods based on state space transformation. The methods using differentiation are the substitution of the constraint equations by derivatives, the Baumgarte–Method and the Pantelides–Algorithm with the use of Dummy Derivatives. Furthermore two methods using projection are considered: the orthogonal projection method and the symmetric projection method. The next approach uses a local coordinate transformation to reduce the index. Lastly the Gear–Gupta–Leimkuhler formulation is considered. At the end the advantages and disadvantages of all these methods are discussed and a basic outline on the functionality and the requirements for the implementation of each method is given.

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1. INTRODUCTION

In general the numerical solution of differential–algebraic equation systems with high index by conventional solution methods for ordinary differential equations is very complex or may even be impossible. Therefore methods for solving this problem are necessary, which leads to the so–called index reduction. The aim of index reduction is to convert a system of differential–algebraic equations into a system of differential–algebraic equations of lower index or a system of ordinary differential equations.

This paper aims to provide an overview of common regularisation methods that are applicable for differential–algebraic equations which are derived from mechanical systems. Additionally, a classification of these different approaches is made. This classification divides the different approaches into three areas, see Hairer, and Wanner (2002): index reduction with the use of differentiation, stabilization of the numerical solution by projection and (local) transformation of the state space. Three different methods of index reduction with the use of differentiation are considered: replacement of the constraint by its derivative, the Baumgarte–Method and the Pantelides–Algorithm. There are two different methods using projections, called the orthogonal projection method and the symmetric projection method. The idea of the method using transformation of the state space is to obtain a system of ordinary differential equations on a manifold. Additionally the Gear–Gupta–Leimkuhler formulation is considered. According to the classification each approach is presented and explained in detail. In conclusion the advantages and disadvantages of the different regularisation methods are discussed.

2. BASIC DEFINITIONS

This section provides a collection of fundamental defintions, which will be needed subsequently. A differential–algebraic equation (abbreviated DAE), see Kunkel, and Mehrmann (2006), is given by an implicit equation

$$F(t, x, \dot{x}) = 0 \tag{1}$$

with a function $F\colon I \times D_x \times D_{\dot{x}} \to \mathbb{R}^n$, where $I \subseteq \mathbb{R}$ is a real interval and D_x, $D_{\dot{x}} \subseteq \mathbb{R}^n$ are open sets, $n \in \mathbb{N}$ and $x\colon I \to \mathbb{R}^n$ is a differentiable function, where $\dot{x}$ is the derivative of x with respect to t. According to the implicit function theorem F can be solved for $\dot{x}$ if the matrix $\frac{\partial F}{\partial \dot{x}}$ is regular.

Variables and equations can be categorised with the following definitions:

- algebraic variable: no derivatives of an algebraic variable may occur in the DAE.
- differential variable: derivatives of a differential variable occur in the DAE.
- algebraic equation: an algebraic equation is an equation where no derivatives occur.
- differential equation: a differential equation is an equation where derivatives occur.

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The algebraic equations of the differential–algebraic equation system $F(t, x, \dot{x}) = 0$ are of the form

$$g(x) = 0, \tag{2}$$

where $g\colon \mathbb{R}^n \to \mathbb{R}^m$ is a function with $m < n$, and they are called constraints or constraint equations.

A differential–algebraic equation has differential index $k \in \mathbb{N}_0$ (see Hairer, and Wanner (2002)) if k is the minimal number of derivatives, so that an ODE can be extracted from the system

$$F(t, x, \dot{x}) = 0,\ \frac{\mathrm{d}F(t, x, \dot{x})}{\mathrm{d}t} = 0, \ldots, \frac{\mathrm{d}^k F(t, x, \dot{x})}{\mathrm{d}t^k} = 0. \tag{3}$$

This generated ODE can (by algebraic transformations) be written in the form $\dot{x} = \varphi(t, x)$ with a function $\varphi\colon I \times D_x \to \mathbb{R}^n$. In the following the differential index will sometimes be only called index.

3. DAES OF INDEX THREE

A general differential–algebraic equation system of index 3 is given as

$$\begin{aligned} \dot{y} &= f(y, z) \\ \dot{z} &= k(y, z, u) \\ g(y) &= 0, \end{aligned} \tag{4}$$

where $f\colon U \times V \to \mathbb{R}^n$, $k\colon U \times V \times W \to \mathbb{R}^m$, $g\colon U \to \mathbb{R}^p$ and $U \subset \mathbb{R}^n$, $V \subset \mathbb{R}^m$, $W \subset \mathbb{R}^p$ open. The variables y and z are differential variables and u represents an algebraic variable. Differentiation of $g(y) = 0$ with respect to t gives

$$g_y \dot{y} = g_y f = 0. \tag{5}$$

The second derivative of $g(y) = 0$ with respect to t yields

$$\begin{aligned} & f^T \otimes g_{yy} \otimes f + g_y f_y \dot{y} + g_y f_z \dot{z} = \\ & \quad = f^T \otimes g_{yy} \otimes f + g_y f_y f + g_y f_z k = 0, \end{aligned} \tag{6}$$

where $\otimes$ stands for the tensor product. Differentiating $g(y) = 0$ three times with respect to t results in a term containing $g_y f_z k_u \dot{u}$. If $g_y f_z k_u$ can be inverted, the given DAE has index 3 and the DAE

$$\begin{aligned} \dot{y} &= f(y, z) \\ \dot{z} &= k(y, z, u) \\ f^T \otimes g_{yy} \otimes f + g_y f_y f + g_y f_z k &= 0 \end{aligned} \tag{7}$$

is of index 1. If $g_y(y) f_z(y, z) k_u(y, z, u)$ can be inverted, $g_y(y) f_z(y, z) k(y, z, u) = 0$ can be solved for u with the use of the implicit function theorem, i.e. there exists a function $h\colon \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^p$ with $u = h(y, z)$.

In conclusion we can say that if $g_y(y) f_z(y, z) k_u(y, z, u)$ is invertible, the DAE (4) has index 3 and the algebraic variable u can be extracted from the second derivative of the constraint $g(y) = 0$.

This method is used in several of the approaches described in sections 5 to 7.

4. DAES IN CONNECTION TO MECHANICAL SYSTEMS

In this section the DAE of mechanical systems is derived, where q are the generalised coordinates and $\dot{q}$ the generalised velocities. Additionally the function $L\colon \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$, which depends on t, q and $\dot{q}$, is given. In the following the variational problem

$$\int_{t_1}^{t_2} L\left(t, q(t), \dot{q}(t)\right) \mathrm{d}t \quad \to \quad \min! \tag{8}$$

is considered. The Euler–Lagrange equations are

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0, \quad i = 1, \ldots, n. \tag{9}$$

In the given case of mechanical systems L can be calculated via

$$L = E_{kin} - E_{pot}, \tag{10}$$

where E_{kin} stands for the kinetic energy and E_{pot} for the potential energy, see Hairer, and Wanner (2002).

Therefore the variational problem has the form

$$\int_{t_1}^{t_2} \left(E_{kin} - E_{pot}\right) \mathrm{d}t \quad \to \quad \min!. \tag{11}$$

The next step is to include auxilary conditions or constraint equations. This can be achieved with two approaches:

- Calculus of Variations:
  The Euler–Lagrange equations result in

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial \left(L + \sum_{j=1}^{m} \lambda_j g_j\right)}{\partial q_i} = 0, \quad i = 1, \ldots, n, \tag{12}$$

  where g_j, $j = 1, \ldots, m$ are the constraint equations. These are furthermore added to the equations in 12, therefore a system of $n + m$ equations is obtained. These equations can be transformed into

$$\begin{aligned} & \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \left(E_{kin} - E_{pot}\right)}{\partial \dot{q}_i}\right) - \\ & - \frac{\partial \left(\left(E_{kin} - E_{pot}\right) + \sum_{j=1}^{m} \lambda_j g_j\right)}{\partial q_i} = 0 \end{aligned} \tag{13}$$

  and the constraint equations yield the $(n + 1)^{th}$ till $(n + m)^{th}$ equation.
- Lagrange–Formalism of the Classical Mechanics:
  For the consideration of the constraint equations $g_1(q) = 0, \ldots, g_m(q) = 0$ these are included directly in the function L, i.e.

$$L = E_{kin} - E_{pot} - \lambda_1 g_1 - \ldots - \lambda_m g_m, \tag{14}$$

  where λ_k, $k = 1, \ldots, m$ are Lagrange Multipliers. The Lagrange Multipliers λ_k are added to the generalised coordinates and $\dot{\lambda}_k$ is added to the generalised velocities. Therefore there are $n + m$ generalisied coordinates and velocities respectively instead of n. The Euler–Lagrange equations are

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