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A Closed-Form Approach to Determine the Base Inertial Parameters of Complex Structured Robotic Systems

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Abstract: The dynamic performance and steady-state control errors of many control schemes improve with increasing model accuracy. This paper presents a method to determine the symbolic expressions of the base inertial parameters and the corresponding regressor matrix for models of robotic systems that are linear in the parameters. This is achieved using a transformation based on the row space of the initially rank-deficient observation matrix. Compared to the state-of-the-art methods the proposed approach can handle complex multi-body structures for instance dynamic models with non-collocation of the position and the torque sensors. In addition it applies for general linear parameter models. The procedure of the algorithm is demonstrated considering the double pendulum dynamics as a closed-form example. Furthermore, the performance of the approach is experimentally validated with the 7 degree of freedom medical robot DLR MIRO.

Keywords: Parameter identification, Reduced-order models, Least-squares identification, Robot dynamics

1. INTRODUCTION

To increase the dynamic performance and to minimize the steady-state control error, many modern control schemes rely on model knowledge of the plant. Especially in the field of robotics, model based control strategies, treated among others in [14,20,9,13,1] were often used, including computed torque, adaptive and impedance control. These control schemes require an adequate model to compensate for the dynamics effects (cf. impedance control) or a complete model which represents the rigid body dynamics of the considered robot (cf. computed torque and/or adaptive control). In the case of an adaptive controller as well as for control schemes, where the model parameters have to be a priori known, the model structure and a parameter identification method are crucial [21].

Due to the kinematics of spatial rotations, the inverse-dynamic equations of multi-body systems are strongly non-linear w.r.t. the state variables. It is a well known fact that, deriving the inverse-dynamic model by the Lagrangian formalism (or by means of the iterative Newton-Euler algorithm), the inertial parameters appear linearly [11]; consequently, the identification model can be formulated linearly w.r.t. the unknown parameters. This property allows the identification of the inertial parameters using linear least-squares methods.

When formulating a robot identification model, a challenging property of multi-body systems has to be considered: the inertia of consecutive bodies is coupled via joints, i.e., in general the set of standard inertial parameters (i.e., mass, mass moment first and second order for each body) consists of dependencies. Furthermore, some parameters do not affect the dynamics. In the case of using the standard inertial parameters, the observation matrix of the identification model is singular and not invertible. Therefore, only the set of identifiable parameters can be estimated, which correspond to non-zero and linearly independent columns of the observation matrix.

To calculate the minimal set of identifiable parameters, which are often referred to as base inertial parameters (BIP), several algorithms have been proposed [12,3–6,10,17]. Most of these algorithms use a manual search strategy which can be applied to a special class of robot structures and aim to derive the identifiable parameters in symbolic form. In [12,4–6,17] the structure of the multi-body system is analyzed. From the structure of the multi-body system it is deduced which parameters do not affect the model output and can be cancelled out and which parameters consist of dependencies and can be grouped. By the mentioned method several algorithms are derived to calculate the BIP of robot structures, where only single degree of freedom joints connect apparent bodies. Two numerical approaches to calculate the BIP have been proposed in [3]. One of the methods is based on singular value decomposition (SVD) and the other method is based on QR decomposition. Both methods are applicable for a large class of mechanical systems and straightforward to implement. The set of BIP is obtained in numeric form. The SVD approach given by [3] can be extended to obtain the BIP in symbolic form. Therefore, the work in [10] applies the SVD approach to get the weighting parameters, numerically. Then a search algorithm based on physical dimensional analysis is used to find corresponding symbolic expressions. Therefore also the structure of the dynamic model has to be taken into account.

In this paper, we present a general algorithm to derive the symbolic expressions of a unique set of identifiable parameters and the corresponding reduced regressor matrix of general linear parameter models. We propose to transform the unknown parameter vector into the row

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space of the initially rank deficient observation matrix. This leads to a minimum set of identifiable parameters, fully describing the model, and thus to minimum degrees of freedom in the solution of the linear least-squares problem. Regarding the exemplified derivation of the set of BIP of robots, the non-singular linear row space transformation is derived symbolically, without analyzing the structure of the model, in contrast to [12,4–6,17]. Furthermore the method applies to complex multi-body structures with multi-degree-of-freedom joints. In addition, as exemplified in the experimental part, the method can be applied to dynamic models with non-collocated placement of the position and torque sensors. Therefore, the proposed closed-form approach is more general than the methods in [12,4–6,10,17]. In contrast to the numeric approaches [3], the output of our algorithm is the identification model in symbolic form. This is an advantage, since the resulting model can be further used, for instance, to find optimally exciting observations for the identification procedure, e.g. robot trajectories [19,21] or to reduce the computational costs to compute the model, e.g. the inverse dynamics of the robot (cf. derivation of BIP).

The paper is organized as follows: In Section 2.2 we present the method to determine the identifiable parameter model. The procedure of the closed form algorithm is exemplary demonstrated for the double pendulum dynamics in Section 3. In Section 4, we validate the performance of the method in experiments with a complex structured robotic system. A brief conclusion is given in Section 5.

2. IDENTIFIABLE PARAMETER MODEL

In this section, we present a method to derive the symbolic set of identifiable parameters and the corresponding regressor matrix for linear parameter models. Thereby, we consider exemplary the inverse dynamics of multi-body systems. First, we introduce the identification model. Then, we apply a transformation such that the resulting model is linear in the BIP.

2.1 Linear parameter model

The inverse dynamics of a multi-body system can be represented by equations of the form

$$\boldsymbol{\Gamma}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}},\boldsymbol{\zeta}) = \boldsymbol{M}(\boldsymbol{q},\boldsymbol{\zeta})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}},\boldsymbol{\zeta})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q},\boldsymbol{\zeta}). \quad (1)$$

Herein $\boldsymbol{q} \in \mathbb{R}^m$ are the m generalized coordinates and $\boldsymbol{M} \in \mathbb{R}^{m\times m}$, $\boldsymbol{C} \in \mathbb{R}^{m\times m}$ and $\boldsymbol{g} \in \mathbb{R}^m$ are inertia matrix, Coriolis/centrifugal matrix and gravity vector, respectively.

The inertial properties of the ith body are determined by the inertial tensor

$$\boldsymbol{\Phi}_i = \begin{bmatrix} XX_i & XY_i & XZ_i \\ & YY_i & YZ_i \\ \text{sym.} & & ZZ_i \end{bmatrix}, \quad (2)$$

the vector of mass moments first order

$$\boldsymbol{MS}_i = [mX_i, mY_i, mZ_i]^T \quad (3)$$

and mass moment zeroth order m_i. These parameters can be summarized for each body in

$$\boldsymbol{\zeta}_i = [XX_i, XY_i, XZ_i, YY_i, YZ_i, ZZ_i, \\ mX_i, mY_i, mZ_i, m_i]^T \quad (4)$$

and for n_b bodies in

$$\boldsymbol{\zeta} = \left[\boldsymbol{\zeta}_1^T, \boldsymbol{\zeta}_2^T, \ldots, \boldsymbol{\zeta}_{n_b}^T\right]^T. \quad (5)$$

The vector $\boldsymbol{\zeta}$ is known as standard inertial parameter vector of the multi-body system.

The inverse dynamic model (1) is linear w.r.t. the standard inertial parameters [11] and can be rewritten as

$$\boldsymbol{\Gamma}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}},\boldsymbol{\zeta}) = \frac{\partial \boldsymbol{\Gamma}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}},\boldsymbol{\zeta})}{\partial \boldsymbol{\zeta}}\boldsymbol{\zeta} = \boldsymbol{X}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}})\boldsymbol{\zeta}. \quad (6)$$

2.2 Linear parameter identification for non-linear systems

In order to estimate $\boldsymbol{\zeta}$ from measurements of $\boldsymbol{q}(t)$, $\dot{\boldsymbol{q}}(t)$, $\ddot{\boldsymbol{q}}(t)$ and $\boldsymbol{\Gamma}(t)$, an overdetermined system of linear equations,

$$\boldsymbol{W}\boldsymbol{\zeta} = \boldsymbol{Y} \quad (7)$$

can be considered. Thereby, the observation matrix

$$\boldsymbol{W} = \left[\boldsymbol{X}(\boldsymbol{x}(t_1))^T, \boldsymbol{X}(\boldsymbol{x}(t_2))^T, \ldots, \boldsymbol{X}(\boldsymbol{x}(t_k))^T\right]^T, \quad (8)$$

is a stacked matrix, where $t_i \in \mathbb{R}$ is time and $\boldsymbol{x} : \mathbb{R} \to \mathbb{R}^c$ are c independent variables (e.g., generalized coordinates and their time derivatives), the regressor matrix $\boldsymbol{X}$ consists of non-linear functions mapping from $\mathbb{R}^c$ into $\mathbb{R}^{m\times n}$, and $\boldsymbol{\zeta} \in \mathbb{R}^n$ is the parameter vector. Similarly,

$$\boldsymbol{Y} = \left[\boldsymbol{y}(t_1)^T, \boldsymbol{y}(t_2)^T, \ldots, \boldsymbol{y}(t_k)^T\right]^T \quad (9)$$

is a stacked vector of dependent variables $\boldsymbol{y} \in \mathbb{R}^m$ (system outputs). In the case of identifying an inverse dynamic model of the form (1), $\boldsymbol{x} = (\boldsymbol{q}^T, \dot{\boldsymbol{q}}^T, \ddot{\boldsymbol{q}}^T)^T \in \mathbb{R}^{c=3m}$ is a set of the generalized joint coordinates and their time derivatives and $\boldsymbol{y} = \boldsymbol{\Gamma}$ are measured joint torques.

If the number of observations k is chosen sufficiently large, i.e., $km \geq n$, a unique (least-squares) solution exists when $\text{rank}(\boldsymbol{W}) = n$. In general for identification models, which are linear in the standard inertial parameters, the observation matrix is rank deficient, i.e., $\text{rank}(\boldsymbol{W}) < n$. Therefore, the goal is to find the reduced regressor matrix $\bar{\boldsymbol{X}}(\boldsymbol{x})$ and corresponding identifiable parameters $\bar{\boldsymbol{\zeta}}$, such that the resulting stacked matrix $\bar{\boldsymbol{W}}$ has full rank. The basic idea is to transform the vector of unknown parameters $\boldsymbol{\zeta}$ of the linear system (7) to the row space of the matrix $\boldsymbol{W}$.

Theorem 1. Consider the linear system of equations (7) with $\boldsymbol{W} \in \mathbb{R}^{km\times n}$ defined by (8), where $km \geq n$ and $r = \text{rank}(\boldsymbol{W}) < n$. Then, there exists a matrix $\boldsymbol{B}$ with generalized pseudo-inverse

$$\boldsymbol{B}^\dagger = \boldsymbol{S}\boldsymbol{B}^T(\boldsymbol{B}\boldsymbol{S}\boldsymbol{B}^T)^{-1} \quad (10)$$

and metric $\boldsymbol{S}$ such that

$$\bar{\boldsymbol{W}}\bar{\boldsymbol{\zeta}} = \boldsymbol{Y}, \quad (11)$$

where

$$\bar{\boldsymbol{W}} = \begin{bmatrix} \bar{\boldsymbol{X}}(\boldsymbol{x}(t_1)) \\ \bar{\boldsymbol{X}}(\boldsymbol{x}(t_2)) \\ \vdots \\ \bar{\boldsymbol{X}}(\boldsymbol{x}(t_k)) \end{bmatrix} \in \mathbb{R}^{km\times r}, \quad (12)$$

$$\bar{\boldsymbol{X}}(\boldsymbol{x}) = \boldsymbol{X}(\boldsymbol{x})\boldsymbol{B}^\dagger, \quad (13)$$

can be uniquely solved for $\bar{\boldsymbol{\zeta}}$ in a least-square sense. Thereby,

$$\bar{\boldsymbol{\zeta}} = \boldsymbol{B}\boldsymbol{\zeta}, \quad (14)$$

represent the identifiable parameters. Additionally, due to this particular choice of $\boldsymbol{B}$, to be equal to the non-zero rows of the reduced-row echelon form $\boldsymbol{E}_W$ of $\boldsymbol{W}$, $\boldsymbol{B}$ is unique.