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Low Frequency Correction of a Multi-degrees-of-freedom Model for Hydraulic Pipeline Systems

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Abstract: For hydraulic pipeline systems, a multi-degrees-of-freedom model is developed from the modal decomposition of the transfer function between flow rate excitation and pressure response. Eigenvectors are taken from the undamped case. Natural frequencies and damping ratios are calculated from modal approximations of the individual pipelines with single frequency approximations at the pipeline system resonances in the low frequency range. The multi-degrees-of-freedom pipeline system model is rebuilt from its modal description and evaluated for a network that connects a pump with two closed volumes.

Keywords: Hydraulic pipelines, Approximate analysis, Eigenmode analysis, Eigenvalues, Eigenvectors, Transfer functions

1. INTRODUCTION

Undamped fluid flow in a pipeline is described by the wave equation, which Ingard (1988) treats as a common model for electromagnetic waves on a cable, sound waves in a fluid, longitudinal waves on a solid bar, and torsional waves on a rod. If damping is taken into account, the model for oil hydraulic applications becomes more specific. It can be based on laminar flow conditions since turbulence would increase pressure loss and is usually avoided in oil hydraulics. The transient laminar flow of a compressible Newtonian fluid in a straight circular pipeline was described by transcendental transfer functions from D'Souza et al. (1964) and has further been modelled by rational fraction modal approximations. Such models were published by Almondo et al. (2006), Ayalew et al. (2005), Hsue et al. (1983), Mäkinen et al. (2000), van Schothorst (1997), and Yang et al. (1991); they can be used for time-domain simulation and are well suited to study the dynamic behaviour of individual pipelines. Compared to the transcendental model by D'Souza et al. (1964), Kojima et al. (2002) encountered large errors when they combined modal approximations for the simulation of compound pipeline systems; they therefore suggested to calculate transcendental transfer functions of the entire system and approximate the result in a second step.

For a closed-end pipeline, injected flow rate excitations and resulting pressure responses, Mikota (2013) derived the modal decomposition of the transcendental pipeline model. Transcendental modal transfer functions were approximated by rational fraction expressions, which lead to a multi-degrees-of-freedom description of the pipeline. Mikota (2014) used this model to investigate a specific pipeline network and experienced similar problems as Kojima et al. (2002). However, by comparing transcendental and approximated transfer functions of the network, it became clear that these problems were located in the low frequency range. They were explained by the fact that for damped pipeline systems, the modal approximation of an individual pipeline is rather inaccurate in the frequency range below the first pipeline resonance. If the pipeline becomes part of a network, this frequency range will contain one or more network resonances, for which the approximation will be wrong.

In this paper, the modal approximations from Mikota (2013) are modified in a way that corrects the low frequency errors for a predefined pipeline system. Proportional damping is enforced on the pipeline system model so that all eigenvectors can be taken from the undamped case. In the low frequency range, natural frequencies and damping ratios are calculated from single frequency approximations of the individual pipelines. Higher natural frequencies and the respective damping ratios are taken from the viscous damping approximaton as used by Mikota (2014). The new method is applied to the pipeline network from Mikota (2014) and leads to a significant improvement of the multi-degrees-of-freedom pipeline system model.

2. EXAMPLE SETUP AND PREVIOUS RESULTS

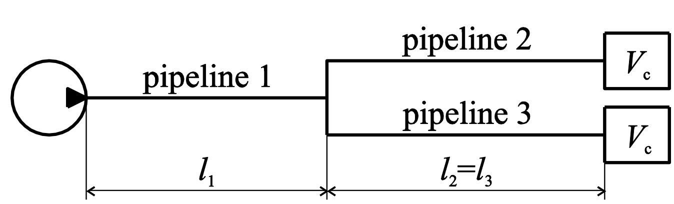

Fig. 1. Hydraulic pipeline network.

To motivate the necessity of a low frequency correction, example and results from Mikota (2014) are summarized in

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this Section. Figure 1 shows the pipeline network under consideration. It consists of pipeline 1 with l_1 = 2.3 m, which is connected to a pump, and pipelines 2 and 3 with $l_2 = l_3$ = 3.0 m, each of which leads to a closed volume with V_c = 1 dm^3 (e.g. a cylinder volume). The inner radius of all pipelines equals r = 1 cm. The fluid bulk modulus is taken as $E = 2\cdot10^9$ Pa and the fluid density as ρ = 1000 kg·m^{-3}, leading to a speed of sound $c = \sqrt{E/\rho} = 1414$ m·s^{-1}; the kinematic viscosity is taken as $\nu = 5\cdot10^{-5}$ m^2·s^{-1}.

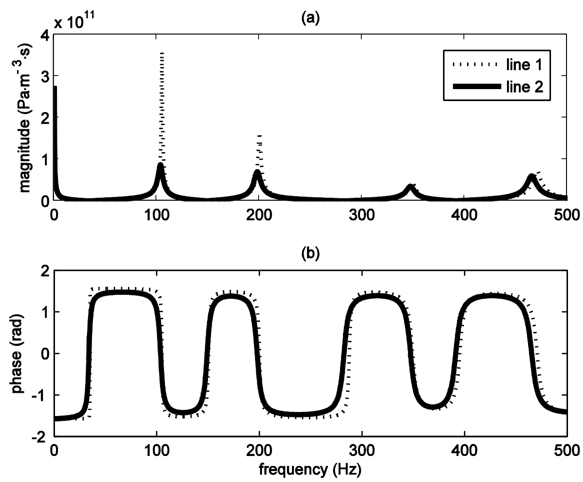

Fig. 2. Comparison of transfer functions between flow rate excitation and pressure response at the inlet of pipeline 1. (a): magnitude, (b): phase. line 1: approximated (uncorrected multi-degrees-of-freedom), line 2: transcendental.

The pump injects a defined flow rate excitation at the inlet of pipeline 1. It therefore makes sense to consider the transfer function between flow rate excitation and pressure response. By a comparison of transcendental and approximated transfer functions, Fig. 2 shows how an uncorrected multi-degrees-of-freedom approximation exaggerates the magnitudes at the lower two resonances. A linear amplitude scale is used to demonstrate the extent of the problem.

3. HYDRAULIC PIPELINE SYSTEM MODEL

Compared to Mikota (2014), the hydraulic pipeline system model is rebuilt from a modal description in which some natural frequencies and damping ratios are corrected. Although the underlying multi-degrees-of-freedom model of an individual pipeline features proportional damping, this is not necessarily the case for the assembled multi-degrees-of-freedom model of the hydraulic pipeline system. To keep within the framework of proportional damping, the eigenvectors of the corrected system are assumed to be real and can therefore be taken from the undamped version of the pipeline system model.

3.1 Undamped Case

For an undamped pipeline with closed ends, the modal decomposition of the transfer function between the flow rate excitation Q_{ex} at the axial coordinate x_k and the pressure response P at the axial coordinate x_j reads

$$\frac{P(x_j,s)}{Q_{ex}(x_k,s)} = \frac{E}{Als} + \frac{2E}{Al}\sum_{n=1}^{\infty} G_n(s)\cos\left(\frac{n\pi x_j}{l}\right)\cos\left(\frac{n\pi x_k}{l}\right) \tag{1}$$

with the modal transfer function

$$G_n(s) = \frac{s}{s^2 + \left(\frac{n\pi c}{l}\right)^2}, \tag{2}$$

where l denotes the length of the pipeline, A is the pipeline cross-sectional area, E is the bulk modulus of the fluid, and c is the speed of sound.

The mobility function of a mechanical system is defined as the frequency response function between excitation force and velocity response. For undamped and proportionally damped systems, Ewins (2000) derives the description of the mobility function in terms of eigenvalues and mass-normalized eigenvectors.

In the following, flow rate excitation and pressure are considered at the discrete coordinates $x_1, x_2, \ldots, x_{N+1}$. If (1) is truncated after mode N, the respective frequency response function can be recognized as mobility function of an undamped mechanical multi-degrees-of-freedom system with mass-normalized $(N+1)\times1$ eigenvectors

$$\boldsymbol{\phi}_0 = \sqrt{\frac{E}{Al}}\cdot\begin{bmatrix}1 & 1 & \ldots & 1\end{bmatrix}^T \tag{3}$$

and

$$\boldsymbol{\phi}_n = \sqrt{\frac{2E}{Al}}\cdot \left[\cos\left(\frac{n\pi x_1}{l}\right) \quad \cos\left(\frac{n\pi x_2}{l}\right) \quad \ldots \quad \cos\left(\frac{n\pi x_{N+1}}{l}\right)\right]^T, \tag{4}$$

$n = 1,2,\ldots,N,$

and natural frequencies

$$\omega_n = \frac{n\pi c}{l}, \quad n = 0,1,\ldots,N. \tag{5}$$

Using the $(N+1)\times(N+1)$ matrix

$$\boldsymbol{\Phi} = \begin{bmatrix}\boldsymbol{\phi}_0 & \boldsymbol{\phi}_1 & \ldots & \boldsymbol{\phi}_N\end{bmatrix} \tag{6}$$

and the $(N+1)\times(N+1)$ diagonal matrix $[\omega_n^2]$, if follows from the orthogonality relations for mass-normalized eigenvectors that the mass matrix of the equivalent mechanical model reads

$$\boldsymbol{M} = \left(\boldsymbol{\Phi}^T\right)^{-1}\boldsymbol{\Phi}^{-1}, \tag{7}$$

and the stiffness matrix becomes

$$\boldsymbol{K}_u = \left(\boldsymbol{\Phi}^T\right)^{-1}[\omega_n^2]\boldsymbol{\Phi}^{-1}, \tag{8}$$