ARTICLE IN PRESS

Solid State Electronics xxx (xxxx) xxx–xxx

Contents lists available at ScienceDirect

Solid State Electronics

journal homepage: www.elsevier.com/locate/sse

Distributed parameter modeling to prevent charge cancellation for discrete thickness piezoelectric energy harvester

M. Krishnasamy[a], Feng Qian[b], Lei Zuo[b], T.R. Lenka[a,*]

[a] Microelectronics & VLSI Design Group, Department of Electronics and Communication Engineering, National Institute of Technology Silchar, Assam 788010, India
[b] Energy Harvesting and Mechatronics Research Lab, Virginia Tech, 311 Durham Hall, Blacksburg, VA 24061, United States

ARTICLE INFO

The review of this paper was arranged by Prof. S. Cristoloveanu

Keywords:
Charge cancellation
Distributed parameter
Energy harvester
Piezoelectric
Segmented

ABSTRACT

The charge cancellation due to the change of strain along single continuous piezoelectric layer can remarkably affect the performance of a cantilever based harvester. In this paper, analytical models using distributed parameters are developed with some extent of averting the charge cancellation in cantilever piezoelectric transducer where the piezoelectric layers are segmented at strain nodes of concerned vibration mode. The electrode of piezoelectric segments are parallelly connected with a single external resistive load in the 1st model (Model 1). While each bimorph piezoelectric layers are connected in parallel to a resistor to form an independent circuit in the 2nd model (Model 2). The analytical expressions of the closed-form electromechanical coupling responses in frequency domain under harmonic base excitation are derived based on the Euler–Bernoulli beam assumption for both models. The developed analytical models are validated by COMSOL and experimental results. The results demonstrate that the energy harvesting performance of the developed segmented piezoelectric layer models is better than the traditional model of continuous piezoelectric layer.

1. Introduction

In recent years, energy harvesting from natural resources has become vital and very useful for low power applications around the globe. Analytical and numerical models play an important role for the evaluation and design of energy harvesters. Mechanical energy converted into usable electrical energy by use of piezoelectric [1], electrostatic [2] and electromagnetic [3] devices. Among these, piezoelectric energy harvester is widely used [4,5] for its merits of small size, easy manufacture and implementation. The most common and extensively studied piezoelectric harvester is the cantilever beam based transducer, which is generally composed of a clamped-free substructure beam and continuous piezoelectric layers mounted on it. Moreover, the mechanical strain of piezoelectric layers are introduced by the bending vibration mode of cantilever which creates the electrical charges due to piezoelectric effect. In a clamped-free beam structure, the higher bending vibration modes have zero-strain node except the 1st vibration mode whereas the sign of strains are oppositely changed. As a result the sign of electrical charges are also changed at strain nodes of piezoelectric layers [6]. Therefore, the discrete electrodes/piezoelectric layers are developed in order to avert the charge cancellation due to strain polarity changes.

A lumped parameter model of a cantilever harvester with two segmented piezoelectric (PZT) layers was developed by Zizys et al. [7] for considering strain nodes of the second vibration mode. Results show that the series connected harvester's electrical voltage increased from 6% to 7.2% as compared to single continuous PZT layer (C-PZT) in higher resistive loads.

Kodali et al. [8] reported only the experimental results of fixed-fixed PZT energy harvester whereas, the six discrete PZT layers are in top and one C-PZT layer in bottom of cantilever. Here, the electrical power of each segment is obtained through independent electrical circuit and over all electrically combined. It shows that the total power is increased by 2.5 times of traditional transducer. Solutions of similar models are analytically derived and analyzed for the beam of uniform thickness with segmented only in electrodes of PZT layers [9]. However, separately connecting each PZT layer to an external resistive load fails to drive the total electrical energy from all PZT segments. Furthermore, the harvested power is usually delivered to a single device or stored in one battery. These limitations inspire the idea to derive the total voltage and power from all segmented electrodes by connecting them in parallel with a single external load resistance. The series circuit is not practical for segmented electrodes, since the wires have to be put in the interlayers between the substructure beam and the PZT layer, which is undependable and might lead to delamination. In literature, the traditional PZT energy harvesters are developed using the C-PZT layers in

---

* Corresponding author.
E-mail addresses: krishnasamy@ieee.org (M. Krishnasamy), fengqian@vt.edu (F. Qian), leizuo@vt.edu (L. Zuo), t.r.lenka@ieee.org (T.R. Lenka).

https://doi.org/10.1016/j.sse.2017.12.010
Received 6 May 2017; Received in revised form 28 November 2017; Accepted 20 December 2017

Please cite this article as: Krishnasamy, M., Solid State Electronics (2017), https://doi.org/10.1016/j.sse.2017.12.010

unimorph/bimorph configurations [1–6]. It is observed that the harvested voltages of three discrete PZT layers (2-strain node) are improved by 2.94–12.2 times of traditional energy harvester in 1st lower three vibration modes [10]. However, the theoretical models using distributed parameter modeling of clamped–free cantilever is not available in current literatures [7–10].

A distributed parameter model of a discrete electrode piezoelectric transducer with tip mass was developed and numerically validated by Wang et al. [11], which only responds in the concerned vibration modes in terms of power and bandwidth, other than the fundamental mode. Xiaobiao et al. [12] analyzed the strain nodes of a PZT flag and obtained the power output using a lumped parameter model of segmented PZT layers. Finite element model was also used to identify strain nodes, as well as the optimal resistive load to derive the maximum power from segmented layers [13]. Stewart et al. [14] proposed a micro sized energy harvesting cantilever using individual PZT elements connected in parallel. It is necessary to develop the theoretical models of designed transducer by distributed parameter in order to describe the electromechanical behavior between the discrete PZT layers and the substructure beam, and also accurately predict the output response of transducer.

This paper presents two analytical models of the distributed parameter system for a cantilever PZT harvester, in which three PZT segments are used to consider the 2-strain nodes of 3rd vibration mode. The discrete PZT layers are parallelly connected along with a single resistive load in the first model (Model 1) as shown in Fig. 1(a), while in second model (Model 2), each bimorph PZT segment is connected in parallel to a resistive load, resulting in three independent electrical circuits as shown in Fig. 1(b). Model 1 can predict and analyze the overall electromechanical behavior and electrical responses of all discrete PZT layers w.r.t single vibrant frequency through single resistive load. Similarly, Model 2 can provide information of independent behavior and electrical response of each discrete PZT layer individually at different vibrant frequencies through independent resistive loads.

2. Model development

The motion of uniform thickness cantilever can be represented as a Euler-Bernoulli beam under harmonic base excitation ($w_b(x,t)$) by the partial differential equation as follows [5]

$$\frac{\partial^2 M(x,t)}{\partial x^2} + C_s I \frac{\partial^5 w(x,t)}{\partial x^4 \partial t} + C_a \frac{\partial w(x,t)}{\partial t} + m \frac{\partial^2 w(x,t)}{\partial t^2} = -m \frac{\partial^2 w_b(x,t)}{\delta t^2} - C_a \frac{\partial w_b(x,t)}{\partial t} \tag{1}$$

Here, the $w_b(x,t)$ is the transverse displacement and gives the relation of its position x with time t, base motion ($w_b(x,t)$), air damping coefficient (C_a), strain rate damping factor (C_s) are defined. The full length of cantilever is L, m is the mass per unit length, and the internal bending moment ($M(x,t)$) is defined for the designed bimorph cantilever as follows [5]

$$M(x,t) = b\left(\int_{-h_p - h_s/2}^{-h_s/2} \sigma_p z dz + \int_{-h_s/2}^{+h_s/2} \sigma_s z dz + \int_{h_s/2}^{h_p + h_s/2} \sigma_p z dz \right) \tag{2}$$

where the thickness of substructure and PZT layer, and width of the cantilever are symbolized as h_s, h_p and b. However, σ_p and σ_s are stress components of PZT layer and substructure beam respectively. It is obtained from

$$\sigma_p = Y_p S_p - e_{31} E_3; \quad \sigma_s = Y_s S_s \tag{3}$$

where Y_s, Y_p, e_{31} and E_3 are the respective Young's modulus, piezoelectric constant and electric field intensity. The voltage across the electrode of PZT layer can be related to the electric field intensity, and the internal bending moment which includes the induced voltage by PZT segments can be written [1] as

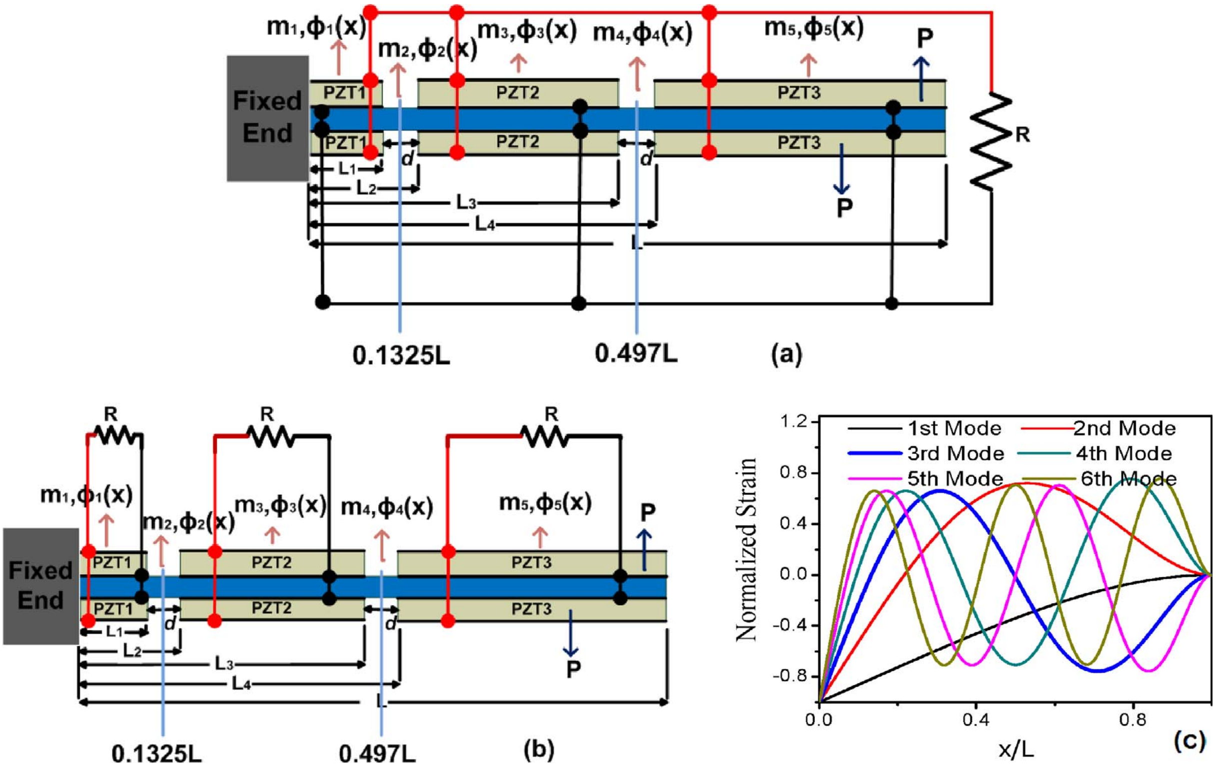

Fig. 1. Cantilevered beam discrete piezoelectric layers energy harvester connected with (a) a single electrical load (Model 1), (b) individual electrical load (Model 2) and (c) the normalized first six modal strains.