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Coordinate-free formation control of multi-agent systems using rooted graphs

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ABSTRACT

This paper studies how to control large formations of autonomous agents in the plane, assuming that each agent is able to sense relative positions of its neighboring agents with respect to its own local coordinate system. We tackle the problem by adopting two types of controllers. First, we use the classical gradient-based controllers on three leader agents to meet their distance constraints. Second, we develop other type of controllers for follower agents: utilizing the properties of rooted graphs, one is able to design linear controllers incorporating relative positions between the follower agents and their neighbors, to stabilize the overall large formations. The advantages of the proposed method are fourfold: (i) fewer constraints on neighboring relationship graphs; (ii) simplicity of linear controllers for follower agents; (iii) global convergence of the overall formations; (iv) implementation in local coordinate systems, in no need of a global coordinate system. Numerical simulations show the effectiveness of the proposed method.

1. Introduction

Distributed coordination of teams of autonomous robots has received increasing attention from the control society in the last decade [1–4], due to the rapid development of computation and communication techniques as well as powerful embedded systems. One typical coordination task is formation keeping, in which a team of mobile agents is required to move collaboratively so that the team manoeuvres as a whole with a prescribed formation shape [5–10]. Formation control of mobile agents, finds various applications in engineering fields, such as sensor networks for data collections [11], unmanned aerial vehicles for military missions [12], and satellite formations for deep space exploration [13].

Various approaches have been proposed to achieve formation stabilization, which can be generally categorized into position-, displacement-, and gradient-based control. Gradient-based control is also as known as distance-based control in some literature. A recent survey [14] of multi-agent formation control discussed the distinctions of the three categories in detail. Position-based control requires that all the agents in a formation-task group are capable of sensing their own positions with respect to a global coordinate system [15–17]. Displacement-based control requires that each mobile agent is equipped with a compass such that all the agents share a common sense of direction [18–21]. Gradient-based formation control, in comparison, only requires that each mobile agent knows relative positions of its neighbors in its own local coordinate [22–27]. To achieve the desired shape by controlling the distances between agents, the interaction graph needs to be rigid or persistent [5,8]. The convergence results of gradient-based formation control only hold locally; in other words, a prescribed formation can be restored only when the agents' shape is close enough to the prescribed one. Global stability analysis has been carried out merely for a class of triangular formations [27–29]. However, the method is difficult to be applied to large formations, since the control laws involving Euclidean distances result in nonlinearity and multiple equilibrium manifolds, which significantly complicates the analysis for formation stabilization. There are some other approaches describe a formation by using bearing measures [30,31].

In this paper, we study formation-keeping tasks in the plane for multiple autonomous agents in their local coordinates. We cope with the challenge by making use of the benchmark case of triangular formations. Existing results [27–29] about triangular formations have proven that under gradient-based control, the convergence to the desired triangular formation is almost global

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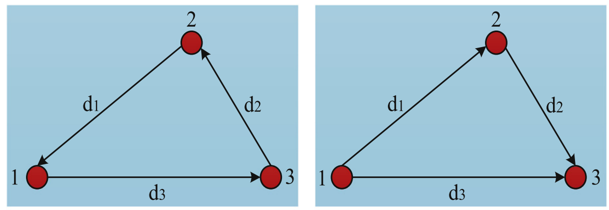

(a) (b)

Fig. 1. Prescribed formation shape. An edge from vertex i to j represents that j can measure the relative position of i. (a) Cyclic triangle. (b) Acyclic triangle.

except for initially collinearly positioned formations. We are able to treat the three agents in the original triangular formation as leaders and then construct large formations by adding more agents. Using the properties of rooted graphs, we design linear controllers incorporating relative positions between the newly-added agents and their neighbors, to stabilize enlarged formations. Compared with the position-based control and displacement-based control methods, our proposed method is coordinate-free, i.e., mobile agents neither need a global coordinate, nor need to share a common sense of direction. Although the controllers are designed differently for the leader agents and for the follower agents, each agent in formation only needs to acquire *relative positions* of its neighbors in its own coordinate system. Compared with the gradient-based control method [5,8,22–25], the advantages of our proposed method are threefold: (i) The convergence of the formation for the whole group of agents is almost *global* except for the case when the positions of the three leaders are initially collinear. (ii) The controllers for the later added agents are linear, which are much simpler than the nonlinear gradient-based formation controllers. (iii) our method requires fewer constraints on the neighbor-relationship graph $\mathbb{G}$, i.e., the graph is not required to be rigid.

The remaining of the paper is organized as follows. In Section 2, we review a class of triangular formation models and the relevant results in the literature. Then, in Section 3, using the properties of rooted graphs, we treat the three agents in the original triangular formation as leaders (i.e., roots) and propose a control method incorporating relative positions between the later added agents and their neighbors, to realize large formations. Furthermore, we prove global convergence for the formations with newly added agents in this section. Numerical examples are given in Section 4 to validate our theoretical analysis. Finally, we make concluding remarks in Section 5.

2. Review on controlling triangular formations

In this section, we review some results in [26–28] about the benchmark case of triangular formations of autonomous agents.

We consider a formation in the plane consisting of three autonomous agents labeled by 1, 2, and 3, shown in Fig. 1. For $i \in \{1, 2, 3\}$, we use $[i]$ to denote $[1] = 2$, $[2] = 3$, and $[3] = 1$. The desired distance between agents i and $[i]$ is d_i; here the d_is are positive numbers and satisfy the triangle inequalities: $d_1 + d_2 > d_3$, $d_2 + d_3 > d_1$, $d_1 + d_3 > d_2$.

Cao et al. in [26–28] have studied how to control three autonomous agents to achieve a prescribed triangular formation. If agent i for $i \in \{1, 2, 3\}$ measures the relative position of agent $[i]$ in its own coordinate system, shown in Fig. 1(a), the agents' dynamics can be described by

$$\begin{aligned}
\dot{x}_1 &= -k(x_1 - x_2)(\|x_1 - x_2\|^2 - d_1^2), \\
\dot{x}_2 &= -k(x_2 - x_3)(\|x_2 - x_3\|^2 - d_2^2), \\
\dot{x}_3 &= -k(x_3 - x_1)(\|x_3 - x_1\|^2 - d_3^2),
\end{aligned} \tag{1}$$

where k is a positive constant to regulate the convergence speed of the formation. For the other case, if agent 2 measures the relative position of agent 1, and agent 3 measures those of agents 1 and 2 in their own coordinates, shown in Fig. 1(b), the agents' dynamics can be written as

$$\begin{aligned}
\dot{x}_1 &= 0, \\
\dot{x}_2 &= k(x_1 - x_2)(\|x_1 - x_2\|^2 - d_1^2), \\
\dot{x}_3 &= -k(x_3 - x_1)(\|x_3 - x_1\|^2 - d_3^2) \\
&\quad + k(x_2 - x_3)(\|x_2 - x_3\|^2 - d_2^2).
\end{aligned} \tag{2}$$

In [27,28], it has been proven that under such gradient-based control laws, system (1) (or (2)) can be stabilized almost globally to an equilibrium corresponding to the triangular formation with the desired shape. Let

$$e_i \triangleq \|x_i - x_{[i]}\| - d_i, \tag{3}$$

for $i = 1, 2, 3$. The desired formation set can be described by

$$\mathcal{E} \triangleq \{x : e_1 = e_2 = e_3 = 0\}.$$

Let $\mathcal{N}$ be the set of points corresponding to the three agent positions which are collinear in the plane. We summarize their main results as follows:

Theorem 1 ([26–28]). *Every trajectory of system (1) (or (2)) starting outside of $\mathcal{N}$, converges exponentially to a finite limit in $\mathcal{E}$.*

In addition, every trajectory of system (1) (or (2)) starting in $\mathcal{N}$ will remain collinear. In practice, collinear positions are easy to become non-collinear because of noise and imprecision in measurements. The equilibria in $\mathcal{N}$ are sensitive to perturbations and thus unstable.

3. Main results: Multiple agent formations

Now we consider the formation task in the plane for $N > 3$ agents. The main idea to accomplish the task is: we choose the three agents of the initial triangular formation to be three leaders for the whole formation and then the other agents join the leaders as followers to realize a large formation. We choose one target configuration that satisfies the prescribed formation shape to be

$$p = [p_1^\top, \dots, p_N^\top]^\top, \tag{4}$$

where $p_i \in \mathbb{R}^2$ represents the position of agent i in some reference coordinate system.

In fact, we adopt two types of controllers for the whole group of agents. First, we control agents 1, 2 and 3 to meet their distance constraints, which have been introduced in Section 2. Second, we design a new type of controller in (8) for the follower agents, which will be presented in this section.

Before going into the details of the controller design, we introduce some useful graph notations [32]. A node v is *k-reachable* from a set $\mathcal{R}$ of nodes if there are k disjoint paths from k different nodes in $\mathcal{R}$ to node v, where every disjoint path contains only one node in $\mathcal{R}$. A directed graph is *k-rooted*, if there exists a subset of k nodes called roots, from which every other node is k-reachable. We take Fig. 2 as an example. Fig. 2(a) is 3-rooted with roots in set $\mathcal{R}$; Fig. 2(b) is not a 3-rooted graph, since node 5 is not 3-reachable from set $\mathcal{R}$.

We use graph $\mathbb{G}$ to describe the neighbor relationship in the formation. In our problem, graph $\mathbb{G}$ satisfies the following condition.

Assumption 1. Suppose that graph $\mathbb{G}$ is 3-rooted and vertices 1, 2, 3 are the three roots of the graph.