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A novel saturated super-twisting algorithm[☆]

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ABSTRACT

In this paper, a feedback control law adopting the super-twisting algorithm is designed such that a continuous and saturated control signal is generated to regulate a first-order system affected by disturbances. For the unperturbed as well as perturbed systems under the proposed control law, global finite-time stability properties are established. In order to indicate the feasibility and effectiveness of the approach, numerical simulations are employed.

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1. Introduction

Having applied a sliding mode control scheme, a closed-loop's satisfactory robust performance is achieved despite the presence of a particular class of plant uncertainties and external disturbances. Traditional sliding mode control, i.e. first-order sliding mode approach, guarantees a saturated and discontinuous control input. A second-order sliding mode technique such as the twisting as well as super-twisting algorithm provides an absolutely continuous control signal in the case that the relative degree of the system with respect to a defined sliding function is one. In general, these high-order sliding mode algorithms improve the sliding accuracy of the conventional sliding mode under discrete-time measurements. They are able to counteract perturbations, which are Lipschitz continuous, and recorded in the literature as the chattering attenuation strategies in the case that the actuator dynamics are fast enough [1,2].

However, for systems with saturating actuators, it is difficult to tune controllers designed based on the aforementioned algorithms such that the control inputs do not exceed given saturation bounds. As a result of going beyond the limits, the windup effect is produced. This is due to the integral actions, which are incorporated therein. Furthermore, the continuous element of the standard super-twisting algorithm (the square root of the state absolute value) will not be within the bounds for any initial condition. In [3], a second-order sliding mode control scheme is introduced, which contributes to a continuous and bounded input. A sub-optimal second-order sliding mode controller is modified in [4] in order to ensure that the sliding variable converges to the origin in a finite time despite the fact that the actuator is saturated. In both of the control laws presented in [3] and [4], high frequency switching between two control strategies based on the saturation bounds may occur. Owing to the limitation on the switching frequency, some undesirable oscillations in the control signals as well as zigzag motions in the system trajectories appear.

A saturated super-twisting algorithm addressing the aforementioned problem is proposed in [5,6]. At most, one switch between two different sliding mode algorithms based on a predefined neighborhood of the origin exists therein. The fairly restrictive assumption, which is made on the bounds of perturbations in [5], is relaxed in [6] applying a disturbance estimator. Furthermore, the convergence of the state to zero is speeded up removing the transient process of the super-twisting algorithm through the estimator. However, this makes the sliding mode control approach redundant since both the estimator and controller reconstruct disturbances. In [7], having applied the standard super-twisting algorithm to the system with bounded control, the largest domain of attraction is computed and the finite-time stability within this domain is proved. It is guaranteed that the control signal remains within the saturation limits and thus there is no windup effect if the initial condition belongs to this domain.

In this paper, a comprehensive saturated super-twisting control law is designed, which is compact in the sense that neither switching from one algorithm to another one nor the estimator is required. Therefore, it is a non-redundant control technique and its implementation enjoys the advantage of a simpler design. Furthermore, the proposed scheme as a global remedy for systems with

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saturating actuators contributes significantly to an improvement in the standard super-twisting performance in the case that the initial values are outside the aforementioned domain of attraction.

The rest of the paper is organized as follows: the problem and the objective are explained in Section 2. The proposed control law design is described in Section 3. The stability analyses of the closed-loop system are carried out in Section 4. Simulation results are demonstrated in Section 5 followed by a conclusion in Section 6.

2. Problem statement

Consider that a system is described by

$$\frac{dz}{dt} = u + a(t), \tag{1}$$

where the output of the system is denoted by $z \in \mathbb{R}$ and u is the scalar control input. Perturbations, which may be functions of time t, represented by $a(t)$ are assumed to be globally bounded and Lipschitz continuous, i.e.

$$|a(t)| \le a_M \quad \text{and} \quad |\frac{da}{dt}| \le L_a, \quad \forall t, \tag{2}$$

where a_M and L_a are some known constants.

The objective is to design a feedback control law for system (1) such that

- the system state z drives to the origin in a finite time despite the presence of disturbances $a(t)$;
- the control signal u is absolutely continuous and saturated by

  $$\sup|u| \le \rho, \quad \forall t, \tag{3}$$

  where ρ is a given constant value.

In order to achieve that, a novel saturated control approach based on a well-known sliding mode algorithm is introduced in the next section.

3. Saturated super-twisting control

Having adopted the super-twisting algorithm, the proposed saturated and continuous actuating signal is obtained through

$$u = -k_1 \mathrm{sat}_\epsilon \left(|z|^{1/2}\right) \mathrm{sgn}(z) + \nu, \tag{4a}$$

$$\frac{d\nu}{dt} = -k_2 \mathrm{sgn}(z) - k_3 \nu, \qquad |\nu_0| \le \frac{k_2}{k_3}, \tag{4b}$$

where the initial value of ν denoted by ν_0 and the positive constants k_1, ϵ, k_2, and k_3 need to be selected appropriately. Sufficient conditions for choosing the control gains are given later. The sat_ϵ function is defined as

$$\mathrm{sat}_\epsilon (y) = \begin{cases} y & \text{for } |y| < \epsilon, \\ \epsilon \ \mathrm{sgn}(y) & \text{for } |y| \ge \epsilon. \end{cases} \tag{5}$$

Therefore, it becomes evident that

$$0 \le \mathrm{sat}_\epsilon \left(|z|^{1/2}\right) \le \epsilon, \quad \forall z. \tag{6}$$

Lemma 1. *If the initial value ν_0 is selected such that $|\nu_0| \le \nu_M = \frac{k_2}{k_3}$ holds, then the control signal u is bounded by*

$$|u| \le k_1 \epsilon + \nu_M, \quad \forall t. \tag{7}$$

Proof. Since (4b) is a linear differential equation of ν with a bounded input, the supremum of $|\nu|$ is

$$\sup|\nu| \le \nu_M, \quad \forall t, \tag{8}$$

where the condition $|\nu_0| \le \nu_M$ is fulfilled. The upper bound of $|u|$ in (7) is derived easily from inequalities (6) and (8). □

Hence, for any initial condition $z(t = 0) = z_0 \in \mathbb{R}$ and $|\nu_0| \le \nu_M$, the absolutely continuous actuating signal remains within the given saturation bounds, i.e. $u \in [-\rho, \rho]$, if and only if the control parameters are chosen such that

$$k_1 \epsilon + \frac{k_2}{k_3} \le \rho \tag{9}$$

is satisfied. For the nominal system, i.e. $a(t) = 0, \ \forall t$, as well as the perturbed system under the proposed control law, the finite-time convergence of the system state z is ensured in the next section setting sufficient conditions for the gains. Two different Lyapunov functions are introduced therein.

4. Stability analysis

For system (1) under control law (4), the closed-loop dynamics is written as

$$\frac{dx_1}{dt} = -k_1 \mathrm{sat}_\epsilon \left(|x_1|^{1/2}\right) \mathrm{sgn}(x_1) + x_2, \tag{10a}$$

$$\frac{dx_2}{dt} = -k_2 \mathrm{sgn}(x_1) - k_3 x_2 + \phi(t), \tag{10b}$$

where a vector is defined as $\boldsymbol{x} = \left[x_1 \ x_2\right]^{\mathrm{T}} = \left[z \ \ \nu + a(t)\right]^{\mathrm{T}}$. Suppose that (2) is satisfied, then $\phi(t) = k_3 a(t) + \frac{da}{dt}$ in the closed-loop system is bounded by

$$|\phi(t)| \le \phi_M = k_3 a_M + L_a, \quad \forall t. \tag{11}$$

4.1. Nominal case

The stability properties of system (10) in the case that the system is not subject to perturbations, i.e. $a(t) = 0, \ \forall t$ and therefore $\phi(t) = 0, \ \forall t$, are investigated in this subsection.

Proposition 1. *In the nominal scenario, the origin $\boldsymbol{x} = \boldsymbol{0}$ is globally finite-time stable if the parameters k_1, ϵ, k_2, and k_3 are positive.*

Proof. Consider the vector

$$\boldsymbol{\zeta} = \left[\zeta_1 \quad \zeta_2\right]^{\mathrm{T}} = \left[|x_1|^{1/2}\mathrm{sgn}(x_1) \quad x_2\right]^{\mathrm{T}}. \tag{12}$$

A strict and Lipschitz Lyapunov function candidate for system (10) without disturbances is introduced here as

$$V_n = \zeta_1^2 + \frac{1}{2k_2}\zeta_2^2 = |x_1| + \frac{1}{2k_2}x_2^2. \tag{13}$$

Remark 1. Since the sat_ϵ function and the linear term $-k_3 x_2$ are incorporated into system (10), it is not possible any more to compute constant matrices for the Algebraic Lyapunov Equation as presented in [8].

The time derivative of the aforementioned Lyapunov function reads as

$$\frac{dV_n}{dt} = -k_1 \mathrm{sat}_\epsilon \left(|x_1|^{1/2}\right) - \frac{k_3}{k_2} x_2^2. \tag{14}$$

It becomes evident that the globally positive definiteness of the radially unbounded function V_n as well as globally negative definiteness of $\frac{dV_n}{dt}$ (according to (6)) is guaranteed by the positive control parameters. Thus, the global asymptotic stability of the origin is achieved.

Lemma 2. *Since the continuous function $\psi = -k_3 x_2 + \phi(t)$ within inhomogeneous system (10) is bounded (due to the boundedness of ν, $a(t)$, and $\phi(t)$), the conditions of the quasihomogeneity principle* [9, Theorem 4.2] *are fulfilled.*

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