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Modified Newton method based iterative learning control design for discrete nonlinear systems with constraints ☆

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ABSTRACT

This paper considers the design of iterative learning control laws for classes of nonlinear dynamics. In particular, a new Newton method design is developed for discrete nonlinear systems in the presence of input constraints, where such constraints will arise in applications. The new design is based on the use of a penalty function and an iterative method for solving an unconstrained nonlinear optimization problem with an algorithm that has monotonic and super linear convergence characteristics. In this new algorithm the input inequality constraints are transformed into equality form by adding auxiliary variables. A cost function is then minimized to produce the new iterative learning control law design. Finally, a simulation based case study is given to illustrate the performance of the new design.

1. Introduction

Many industrial systems are required to undertake the same finite duration task over and over again. In operation, an execution, termed a trial in this paper, is completed over the finite trial length, the system resets to the starting location and the next trial can begin either immediately after the resetting is complete or after a further period of time has elapsed. Once a trial has been completed, all data generated during this trial is available to update the control signal for the next trial and thereby improve performance from trial-to-trial.

Iterative Learning Control (ILC) has been especially developed for such systems. Since the first work [1] it has become an established area of control systems research and application. The survey papers [2,3] give comprehensive overviews of developments up to their years of publication. Major application areas include robotics and various forms of manufacturing processes, see, e.g., [4], and also a transfer from engineering to healthcare for robotic-assisted upper limb stroke rehabilitation with supporting clinical trials [5,6], where the Newton method has also been used [7].

Let the integer k denote the trial number, $y_k(p)$ the output and $u_k(p)$ the input signals on this trial. All signals are defined over the finite interval $0 \leq p \leq N-1$, where $N < \infty$ denotes the number of sampling instants along the trial and in this paper attention is restricted to single-input single-output (SISO) systems with an immediate generalization to the multiple-input multiple-output case. A reference signal, denoted by $y_d(p)$, $0 \leq p \leq N-1$, is assumed to be available. Given this signal, the error on trial k is $e_k(p) = y_d(p) - y_k(p)$ and the basic ILC design problem is to force the sequence $\{e_k\}_{k\geq 0}$ to converge to zero, or to within an acceptable tolerance, in k where convergence is in terms of the norm on the underlying function space.

A large class of model-based ILC laws are designed using optimization, where the gradient method, see, e.g., [8] has been used. However, gradient-based designs may result in slow convergence speed and low efficiency and this performance issue has also led to use of the conjugate gradient method. For nonlinear plant models, a basic Newton method design does not guarantee that a matrix critical to the whole approach is nonsingular.

This problem has led, in the non-ILC literature, to the development of modified quasi Newton methods, such as the BFGS (Broyden, Fletcher, Goldfarb, Shanno) algorithm [9], which given suitable development in the ILC setting, may enable a faster trial-to-trial error convergence rate. Moreover, solving the nonlinear equations defining the entries in the associated Hessian matrix is not required, which greatly reduces the computation and improves the efficiency. Previous work in the non-ILC literature has

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developed a penalty function method for a class of constrained optimization problems together with convergence analysis, see, e.g., [10].

This paper addresses ILC design based on a modified Newton method and the major novel contributions are:

(i) the BFGS optimization algorithm is extended to ILC design for discrete nonlinear systems,
(ii) a design algorithm with monotonic and super-linear (which is a desirable general requirement in applications) trial-to-trial convergence rate where the speed of convergence is a requirement, and
(iii) an extension to design in the presence of control input inequality constraints.

The remainder of this paper is organized as follows. Section 2 introduces a class of discrete nonlinear systems considered and writes the ILC dynamics as a set of discrete nonlinear algebraic equations to provide the starting point for Newton method based design. In Section 3, a design based on the BFGS algorithm is developed. Section 4 formulates and solves the constrained ILC design problem for constraints on the control input on each trial. The solution is based on transforming the constrained problem to equality form by the use of a penalty functions. In Section 5, the monotonic and super linear convergence characteristics of the design are established and Section 6 gives a numerical example to highlight the new results. Finally, the last section summarizes the results obtained and discusses possible future research.

The notation used throughout this paper is standard. In particular, $\Re^n$ denotes the n-dimensional Euclidean space with norm $\|x\| = \sqrt{x^T x}$. A symmetric positive-definite matrix, say Υ, is denoted by $\Upsilon \succ 0$ and I denotes the identity matrix with compatible dimensions.

2. Background and problem formulation

This paper considers single-input single-input discrete nonlinear systems described by the following state-space model in the ILC setting

$$\begin{aligned} x_k(p+1) &= f(x_k(p), u_k(p)), \\ y_k(p) &= h(x_k(p)), \end{aligned} \tag{1}$$

where the nonnegative integer subscript k denotes the trial number, p denotes the sampling instants, $0 \leq p \leq N-1$, $N < \infty$ is the number of samples along the trial (N times the sampling period gives the trial length), $x_k(p) \in \Re^n$, $u_k(p) \in \Re$ and $y_k(p) \in \Re$ represent the system state vector, input and output, respectively, and $f(\cdot)$ and $h(\cdot)$ are vector valued nonlinear functions. Without loss of generality, it is assumed that $x_k(0) = x_d(0)$, i.e., an identical state initial vector on each trial.

The basis of the Newton method for ILC design is to replace the state-space model (1) by a set of algebraic equations in $\Re^N$ and requires the introduction of the input and output time-series vectors

$$u_k = \begin{bmatrix} u_k(0) & u_k(1) & \cdots & u_k(N-1) \end{bmatrix}^T,$$
$$y_k = \begin{bmatrix} y_k(1) & y_k(2) & \cdots & y_k(N) \end{bmatrix}^T.$$

Using (1), the relationships between the input and output time-series can be expressed in terms of the following algebraic functions $g_1, g_2, \ldots, g_N$

$$\begin{aligned} y_k(1) &= h(x_k(1)) = h(f(x_d(0), u_k(0))) = g_1(x_d(0), u_k(0)), \\ y_k(2) &= h(x_k(2)) = h(f(x_k(1), u_k(1))) = g_2(x_d(0), u_k(0), u_k(1)), \\ &\vdots \\ y_k(N) &= h(x_k(N)) = h(f(x_k(N-1), u_k(N-1))) \\ &= g_N(x_d(0), u_k(0), \ldots, u_k(N-1)). \end{aligned}$$

Also, since the state initial vector on each trial is independent of k, the state-space model (1) can be represented by an algebraic function in $\Re^N$ with the structure

$$y_k = g(u_k), \tag{2}$$

where

$$g(u_k) = \begin{bmatrix} g_1(x_d(0), u_k(0)) & g_2(x_d(0), u_k(0), u_k(1)) & \ldots & g_N(x_d(0), u_k(0), \ldots, u_k(N-1)) \end{bmatrix}^T.$$

The general ILC design problem is to find a control input sequence $\{u_k\}$ such that

$$\lim_{k\to\infty} \|e_k\| = 0, \ \lim_{k\to\infty} \|u_k - u_\infty\| = 0,$$

where u_∞ is termed the learned control and $\|\cdot\|$ denotes the norm on the underlying function space. In the case considered, the ILC dynamics have now been formulated as the nonlinear equations (2) and the problem of finding the desired input which forces (1) to track the supplied reference signal y_d is equivalent to finding the solution that satisfies (2) with y_k replaced by the pre-specified reference signal $y_d = \begin{bmatrix} y_d(1) & y_d(2) & \cdots & y_d(N) \end{bmatrix}^T$.

Following the developments in, e.g., [11], the Newton-based ILC law is

$$u_{k+1} = u_k + z_{k+1}, \quad G_k(u_k) z_{k+1} = e_k, \ k \geq 1, \tag{3}$$

where $G_k(u_k)$ is the gradient matrix of $g(u_k)$. This law avoids (potentially) complex calculations to form the inverse of the nonlinear system dynamics (2). The inverse computation has been avoided by introducing $u_{k+1} = u_k + z_{k+1}$, where $z_{k+1} = (G_k(u_k))^{-1} e_k$, which is computed by solving $G_k(u_k) z_{k+1} = e_k$. In ILC terms, introducing $G_k(u_k)$ is equivalent to the linearization of (1) on trial k at (u_k, x_k). It can be shown, using properties of the parallel-chord method for solving nonlinear multivariable equations, see, e.g., [12], that, if convergent a Newton-based method exhibits local quadratic convergence, i.e., for the ILC case [11] the convergence of u_k to u_∞ satisfies

$$\|u_{k+1} - u_\infty\| \leq c \|u_k - u_\infty\|^2, \quad c > 0. \tag{4}$$

As a consequence the Newton-based ILC law (3) has the quadratic convergence property, but slow convergence speed can result. Also some applications require design in the presence of constraints to formulate a physically meaningful control law, see Section 4 for further discussion of an application area where input constraints are particularly required.

To address these issues, this paper develops the standard Newton method based ILC design to obtain a new version that has the super-linear convergence property for applications where fast trial-to-trial convergence is required (without compromising other requirements). Also the design is extended to allow input constraints, where these are particularly relevant in ILC design since this form of control is based on direct computation of the control input for the next trial using previous trial data. The analysis that follows in this paper assumes that $g(\cdot)$ is a twice continuously differentiable function.