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Distributed Kalman filter in a network of linear systems

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ABSTRACT

This paper is concerned with the problem of distributed Kalman filtering in a network of interconnected subsystems with distributed control protocols. We consider networks, which can be either homogeneous or heterogeneous, of linear time-invariant subsystems, given in the state-space form. We propose a distributed Kalman filtering scheme for this setup. The proposed method provides, at each node, an estimation of the state parameter, only based on locally available measurements and those from the neighbor nodes. The special feature of this method is that it exploits the particular structure of the considered network to obtain an estimate using only one prediction/update step at each time step. We show that the estimate produced by the proposed method asymptotically approaches that of the centralized Kalman filter, i.e., the optimal one with global knowledge of all network parameters, and we are able to bound the convergence rate. Moreover, if the initial states of all subsystems are mutually uncorrelated, the estimates of these two schemes are identical at each time step.

1. Introduction

There has been an increasing activity in the study of distributed estimation in a network environment. This is due to its broad applications in many areas, including formation control Subbotin and Smith [1], Lin et al. [2], distributed sensor network Zhang et al. [3] and cyber security Teixeira et al. [4], Zamani et al. [5]. This paper examines the problem of distributed estimation in a network of subsystems represented by a finite dimensional state-space model. Our focus is on the scenario where each subsystem obtains some noisy measurements, and broadcasts them to its nearby subsystems, called *neighbors*. The neighbors exploit the received information, together with an estimate of their internal states, to make a decision about their future states. This sort of communication coupling arises in different applications. For example, in control system security problems Teixeira et al. [4], distributed state estimation is required to calculate certain estimation error residues for attack detection. Similarly, for formation control Lin et al. [6], Zheng et al. [7], Lin et al. [8], each subsystem integrates measurements from its nearby subsystems, and states of each subsystem need to be estimated for distributed control design purposes. The main objective of this paper is to collectively estimate the states of all subsystems within such a network. We will propose a novel distributed version of the celebrated Kalman filter.

The current paper, in broad sense, belongs to the large body of literature regarding distributed estimation. One can refer to Lopes and Ali [9], Kar et al. [10], Conejo et al. [11], Gómez-Expósito et al. [12], Marelli and Fu [13], Olfati-Saber [14], Ugrinovskii [15], Ugrinovskii [16], Zamani and Ugrinovskii [17], Khan and Moura [18], Olfati-Saber [19], He et al. [20] and the survey paper Ribeiro et al. [21], as well as references listed therein, for different variations of distributed estimation methods among a group of subsystems within a network. A consensus based Kalman filter was proposed in Olfati-Saber [14]. The author of Ugrinovskii [15] utilized a linear matrix inequality to minimize a H_∞ index associated with a consensus based estimator, which can be implemented locally. Some of the results there were then extended to the case of switching topology in Ugrinovskii [16]. The same problem was solved using the minimum energy filtering approach in Zamani and Ugrinovskii [17]. The reference [20] proposed an event-based distributed Kalman filter for estimating a common state in a sensor network. A common drawback of the state estimation methods described above is that, being based on consensus, they require, in theory, an infinite number of consensus iterations at each time step. This results in computational and communication overload. To avoid this, in this paper we exploit the network structure to

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achieve a distributed Kalman filter method which requires only one prediction/update step at each time step.

Moreover, it is worthwhile noting that there is a major difference between the above-mentioned works and the problem formulation in the current paper. More precisely, in the former, the aim of each subsystem is to estimate the aggregated state which is common to all subsystems. In contrast, in the problem studied here, each subsystem is dedicated to the estimation of its own internal state, which in general is different from those of other subsystems. This allows the distributed estimation algorithm to be scalable to networked systems with a large number of subsystems where requiring each subsystem to estimate the aggregated state is both computationally infeasible and practically unnecessary.

To show the effectiveness of the proposed algorithm, we compare our method with the classical (centralized) Kalman filter, which is known to be optimal (in the minimum error covariance sense). The classical method requires the simultaneous knowledge of parameters and measurements from all subsystems within the network to carry out the estimation. In contrast, our proposed distributed estimation algorithm runs a local Kalman filter at each subsystem, which only requires the knowledge of local measurements and parameters, as well as measurements from neighbor subsystems. Hence, it can be implemented in a fully distributed fashion. We show that the state estimate, and its associated estimation error covariance matrix, produced by the proposed distributed method asymptotically converge to those produced by the centralized Kalman filter. We provide bounds for the convergence of both the estimate and the estimation error covariance matrix. A by-product of our result is that, if the initial states of all subsystems are uncoupled (i.e., they are mutually uncorrelated), the estimates produced by our method are identical to that of the centralized Kalman filter.

The remainder of the paper is structured as follows. In Section 2, we describe the network setup and its associated centralized Kalman filter. In Section 4, we describe the proposed distributed Kalman filter scheme. In Section 5, we demonstrate the asymptotic equivalence between the proposed distributed filter and the centralized one, and provide bounds for the convergence of the estimates and their associated estimation error covariances. Simulation results that support our theoretical claims are presented in Section 6. Finally, concluding remarks are given in Section 7.

2. System description

In this paper we study networks of N time-invariant subsystems. Subsystem i is represented by the following state-space model:

$$x_{k+1}^{(i)} = A^{(i)}x_k^{(i)} + z_k^{(i)} + w_k^{(i)}, \tag{1}$$

$$y_k^{(i)} = C^{(i)}x_k^{(i)} + v_k^{(i)}. \tag{2}$$

The subsystems are interconnected as follows:

$$z_k^{(i)} = \sum_{j\in\mathcal{N}_i} L^{(i,j)}y_k^{(j)}, \tag{3}$$

where $x_k^{(i)} \in \mathbb{R}^{n_i}$ is the state, $y_k^{(i)} \in \mathbb{R}^{p_i}$ the output, $w_k^{(i)}$ is an i.i.d Gaussian disturbance process with $w_k^{(i)} \sim \mathcal{N}(0, Q_i)$, and $v_k^{(i)}$ is an i.i.d. Gaussian measurement noise process with $v_k^{(i)} \sim \mathcal{N}(0, R_i)$. We further suppose that $\mathcal{E}\left(w_k^{(i)}w_k^{(j)\top}\right) = 0$ and $\mathcal{E}\left(v_k^{(i)}v_k^{(j)\top}\right) = 0$, $\forall i \neq j$ and $\mathcal{E}\left(x_k^{(i)}w_k^{(j)\top}\right) = 0$, $\mathcal{E}\left(x_k^{(i)}v_k^{(j)\top}\right) = 0\ \forall i, j$. We also denote the neighbor set of the subsystem i by $\mathcal{N}_i = \left\{j : L^{(i,j)} \neq 0\right\}$.

Remark 1. We note in (1)–(2) that the coupling between neighboring subsystems is solely caused through the $z_k^{(i)}$ term in'(3). The main motivation for considering such coupling comes from distributed control, where'(1) represents the model of an autonomous subsystem (or agent) with $z_k^{(i)}$ being the control input, and (3) represents a distributed control protocol, which employs feedback only from neighboring measurements. This type of distributed control is not only common for control of multi-agent systems (see, for example, Lin et al. [2], Lin et al. [6], Lin et al. [8], Zheng et al. [7]), but also realistic for large networked systems, since only neighboring information is both easily accessible and most useful for each subsystem.

It is worthwhile noting that the dynamical descriptions (1)–(3) can be regarded as a very general setting for the well-known consensus algorithm [22], i.e., when it is run over a group of interconnected multi-input-multi-output linear subsystems expressed in state space form. Additionally, this model can capture interactions within linear dynamical networks. Interested readers can refer to Zamani et al. [23], Sanandaji et al. [24], Sanandaji et al. [25] and Dankers et al. [26], where the authors exploited a similar model for conducting system identification analysis in linear dynamical networks. Finally, this model turns out to be an effective one for studying properties of networked subsystems [5].

We emphasize that the distributed state estimation problem arises for the networked system (1)–(3) because of our allowance for measurement noises $v_k^{(i)}$ in (2). This consideration is very important for applications because measurement noises are unavoidable in practice. This also sharply distinguishes our distributed control formulation from most distributed control algorithms in the literature, where perfect state measurement is often implicitly assumed.

We define $\xi_k^\top = \left[\left(\xi_k^{(1)}\right)^\top, \ldots, \left(\xi_k^{(I)}\right)^\top\right]$ and $\Xi_k = \{\xi_1, \ldots, \xi_k\}$, where (ξ, Ξ) stands for either (x, X), (y, Y), (z, Z), (w, W) or (v, V); moreover, we denote $\Upsilon = \operatorname{diag}\left\{\Upsilon^{(1)}, \ldots, \Upsilon^{(I)}\right\}$, where Υ stands for either A, B, C, Q or R, and $L = \left[L^{(i,j)} : i, j = 1, \ldots, N\right]$.

Using the above notation, we let the initial state of all subsystems have the joint distribution $x_0 \sim \mathcal{N}(\mu, P)$. We can also write the aggregated model of the whole network as

$$\begin{aligned} x_{k+1} &= Ax_k + LCx_k + w_k + BLv_k \\ &= \tilde{A}x_k + e_k, \end{aligned} \tag{4}$$

$$y_k = Cx_k + v_k, \tag{5}$$

with

$$\tilde{A} = A + LC \quad \text{and} \quad e_k = w_k + Lv_k. \tag{6}$$

It then follows that

$$\operatorname{cov}\left(\begin{bmatrix} e_k \\ v_k \end{bmatrix}\begin{bmatrix} e_k^\top & v_k^\top \end{bmatrix}\right) = \begin{bmatrix} \tilde{Q} & \tilde{S} \\ \tilde{S}^\top & R \end{bmatrix}, \tag{7}$$

where $\tilde{Q} = Q + LRL^\top$ and $\tilde{S} = LR$.

3. Centralized Kalman filter

Consider the standard (centralized) Kalman filter. For all $k, l \in \mathbb{N}$, let

$$\begin{aligned} \hat{x}_{k|l} &\triangleq \mathcal{E}(x_k|Y_l), \\ \Sigma_{k|l} &\triangleq \mathcal{E}\left(\left[x_k - \hat{x}_{k|l}\right]\left[x_k - \hat{x}_{k|l}\right]^\top\right). \end{aligned} \tag{8}$$

Our aim in this subsection is to compute $\hat{x}_{k|k}$ in a standard centralized way. Notice that Eq. (7) implies that, in the aggregated system formed by (1)–(2), the process noise e_k and the measurement noise v_k are mutually correlated. Taking this into account, it follows from [27, S 5.5] that the prediction and update steps of the (centralized) Kalman filter are initialized by $\hat{x}_{0|0} = \mu$ and $\Sigma_{0|0} = P$, and proceed as follows: