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Stability criteria of linear systems with multiple input delays under truncated predictor feedback☆

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ABSTRACT

The model reduction technique as a feedback design approach for linear systems with input delays achieves finite spectrum assignment of the closed-loop system in a delay free form. The resulting predictor-type feedback law contains a distributed term that involves the convolution between the past input and the state transition matrix and hence causes difficulty in its implementation. For the purpose of easy implementation, the truncated predictor feedback (TPF) simplifies the predictor feedback by discarding the distributed term, and delay independent truncated predictor feedback further eliminates the delay-dependent exponential factor. In this paper, we consider the stabilization of a general linear system with multiple time-varying input delays by TPF or delay independent TPF. Stability criteria expressed in terms of input delays and the feedback parameter are derived through Lyapunov–Krasovskii stability analysis, and then numerically studied under both feedback laws.

1. Introduction

Time delay in control input or system state has been widely observed in engineering systems. Examples of time delay are computational time required by control algorithms, transmission lags in remotely controlled plants or ignition delays in diesel engines (see, for example, [1–5] and [6]). Time delay incurs deteriorating performance of the control system in terms of closed-loop stability [7], robustness to disturbances [8] and adaptation to unknown system parameters [9]. As a result, much effort has been devoted to designing control laws that take input delay into account (see [10] and [11]). Many engineering applications involve multiple input delays. For example, the consensus problem in multi-agent systems with distinct communication delays stimulates much application-orientated research to alleviate the effects of multiple input delays (see [12] and [13]).

The model reduction technique, first proposed in [14], achieves active controller design for linear systems with discrete and distributed input delays by introducing the information of delays in a predictor-type feedback. Successful attempts have been made in [15] to extend the model reduction technique for a class of nonlinear systems with state delays through Lyapunov function-based controller design. Further development of the results of [16] was made in [17] to deal with a class of linear systems with state delays through recursively using forwarding and backstepping finite spectrum assignment.

A typical predictor feedback law derived from the model reduction technique [16] for a linear system with a single input delay τ can be expressed as

$$u(t) = Fx(t+\tau) = Fe^{A\tau}x(t) + F\int_{t-\tau}^{t} e^{A(t-s)}Bu(s)\mathrm{d}s,\ \ t \geq 0,$$

where x, u, A and B are the state vector, input vector, state matrix and input matrix of the system, respectively, and F is a feedback gain matrix such that $A+BF$ is Hurwitz. It can be seen that the feedback law consists of a linear state feedback term and a distributed term that involves the convolution of the state transition matrix and the past input. The resulting closed-loop system is free of delay and is asymptotically stable. However, such a predictor feedback law incurs considerable difficulty to digital implementation due to the distributed term. To overcome this difficulty, [18] proposed to discard the distributed term in the predictor feedback and obtain a simplified linear feedback law. Such a feedback law is referred to as the truncated predictor feedback (TPF) and would asymptotically stabilize linear systems that are not exponentially unstable for an arbitrarily large delay as long as the feedback gain matrix is designed by using the low gain feedback technique [19]. Moreover, the truncated predictor feedback law without the presence of $e^{A\tau}$ formulates a delay independent linear feedback which improves robustness to time-varying delays or even unknown delays. This delay independent TPF law also achieves asymptotic stabilization for an arbitrarily large delay when the poles of the open loop

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system are in the open left-half plane or at the origin. The eigenstructure assignment based low gain feedback design technique was used in [18] to construct the feedback gain matrix, and an alternative approach, referred to as the parametric Lyapunov equation based low gain feedback design [20], has been demonstrated in [21] to achieve exactly the same stabilizing goal as in [18].

To explore the applicability of this new feedback design technique for constructing the truncated predictor feedback law to general linear systems with a single input delay, [22] extends the results of [21] to exponentially unstable systems and showed that delay bound under which the truncated predictor feedback achieves asymptotic stability of the closed-loop system is inverse proportional to the sum of all the exponentially unstable poles of the system. On the other hand, [23] considered stabilization of general linear systems that are allowed to have exponentially unstable poles via delay independent truncated predictor feedback law. A delay bound was established through Lyapunov–Krasovskii stability analysis. The counterparts of [22] and [23] to the discrete-time setting were also developed in [24] and [25], respectively.

Considering the success of the Lyapunov equation based feedback design to stabilizing linear systems with a single input delay, [26] studied the problem of stabilizing linear systems with multiple input delays via both the truncated predictor feedback and delay independent truncated predictor feedback. In particular, systems without exponentially unstable open loop poles would be asymptotically stabilized by the truncated predictor feedback for arbitrarily large delays as long as the feedback parameter is tuned small enough. Furthermore, the delay independent truncated predictor feedback would also asymptotically stabilize systems with all open loop poles in the open left-half plane or at the origin.

In this paper, we consider the problem of stabilizing a general linear system with multiple time-varying input delays by truncated predictor feedback and delay independent truncated predictor feedback. In particular, we first extend a result of the truncated predictor feedback stabilization in [26] to general linear systems that may be exponentially unstable. A stability criterion in terms of a scalar inequality that involves the information of input delays and feedback parameter is derived. Simplification of this stability criterion to the class of systems without exponentially unstable poles shows that the truncated predictor feedback law with a sufficiently small feedback parameter would asymptotically stabilize the system for arbitrarily large delays. This observation is consistent with the result of [26]. On the other hand, interpretation of the stability criterion to the class of systems with only a single input delay leads to a result consistent with those in [22] and [21]. Next, we extend a result of delay independent truncated predictor feedback stabilization in [26] to general linear systems that may be exponentially unstable. An upper bound for all the input delays is proposed under which the asymptotic stability of the closed-loop system is guaranteed. Examination of this upper bound in the case of systems with all open loop poles in the open left half plane or at the origin shows that the delay independent truncated predictor feedback law would stabilize the system for arbitrarily large delays as long as the feedback parameter is chosen small enough, which again coincides with the result of [26].

The organization of the paper is given as follows. The problems of stabilizing general linear systems with multiple input delays via truncated predictor feedback and delay independent truncated predictor feedback are formulated in Section 2, where the explicit formulas of the two feedback laws are given through the use of the model reduction technique and delay elimination. Some technical lemmas that are necessary for our stability analysis are presented. Section 3 and Section 4 provide the stability analysis under TPF and delay independent TPF, respectively. Numerical analysis and simulation results are given in Section 5 to illustrate our theoretical results. Section 6 concludes the paper.

Notation: Throughout the paper, we use rather standard notation. The set of real numbers, positive numbers and natural numbers are denoted as $\mathbb{R}$, $\mathbb{R}^+$ and $\mathbb{N}$, respectively. Also, $\mathbf{I}[a, b]$ represents the set of all integers between $a \in \mathbb{N}$ and $b \in \mathbb{N}$ such that $a \le b$. For a vector $v \in \mathbb{R}^{n_v}$, $\|v\|$ stands for its Euclidean norm, and for a matrix $M \in \mathbb{R}^{n_M \times n_M}$, $\|M\|$ denotes the induced matrix norm from the Euclidean norm.

2. Problem statement and preliminaries

We consider the stabilization of a linear system with multiple time-varying input delays,

$$\begin{cases} \dot{x}(t) = Ax(t) + \displaystyle\sum_{i=0}^{N} B_i u(t - \tau_i(t)), \quad t \ge 0, \\ x(t) = \psi(t), \ t \in [-D, 0], \end{cases} \tag{1}$$

where $A \in \mathbb{R}^{n\times n}$ and $B_i \in \mathbb{R}^{n\times m}$, $i \in \mathbf{I}[0, N]$, are state matrix and input matrices, respectively, $N + 1 \in \mathbb{N} \setminus \{0\}$ is the number of distinct input delays $\tau_i(t)$, $i \in \mathbf{I}[0, N]$, and $D \in \mathbb{R}^+$ is an upper bound for all the input delays, *i.e.*, $\tau_i(t) \in [0, D]$, $\forall i \in \mathbf{I}[0, N]$, $\forall t \ge 0$. The initial condition of the delayed system is given by $x(t) = \psi(t)$, $t \in [-D, 0]$. It is assumed that both $\tau_i(t)$, $t \ge 0$, $i \in \mathbf{I}[0, N]$ and $\psi(t)$, $t \in [-D, 0]$, are continuous functions of t, which collectively guarantee the existence and uniqueness of a continuously differentiable solution on $t > 0$ for the closed-loop system under any linear state feedback law.

Construction of a truncated predictor feedback law and a delay independent truncated predictor feedback law for system (1) based on the model reduction technique and delay elimination is now recalled from [26]. We define an auxiliary signal as

$$\phi(t) = x(t) + \sum_{i=0}^{N} \int_{t-\tau_i}^{t} \mathrm{e}^{A(t-\tau_i-s)} B_i u(s) \mathrm{d}s, \tag{2}$$

where, for the purpose of simplicity, all delays are assumed to be constant. Then, the time derivative of $\phi(t)$ along the trajectory of system (1) is given by $\dot{\phi}(t) = A\phi(t) + Bu(t)$ with $B = \sum_{i=0}^{N} \mathrm{e}^{-A\tau_i} B_i$. If the pair of (A, B) is controllable, there exists a feedback gain matrix F such that $A + BF$ is Hurwitz. Under the feedback law

$$u(t) = F\phi(t) = Fx(t) + F \sum_{i=0}^{N} \int_{t-\tau_i}^{t} \mathrm{e}^{A(t-\tau_i-s)} B_i u(s) \mathrm{d}s, \tag{3}$$

the closed-loop system $\dot{\phi}(t) = (A + BF)\phi(t)$ is asymptotically stable. Since $\lim_{t\to\infty} \phi(t) = 0$, it follows from (3) that $\lim_{t\to\infty} u(t) = 0$, which further implies that $\lim_{t\to\infty} x(t) = 0$ by virtue of (2).

The control law (3) is typically referred to as predictor feedback since its simplified version for some classes of systems has the form of the prediction of future state. Note that the second term of the predictor feedback involves the convolution of the state transition matrix with the past input. Also, both sides of (3) contain input u at time instance t, which may cause the digital implementation of the controller to fail [27]. If the second term in the predictor feedback law is dropped, a simplified linear feedback law is formulated as $u(t) = Fx(t)$, namely, the truncated predictor feedback. The feedback gain matrix can be constructed as $F = F(\gamma) = -B^{\mathrm{T}} P(\gamma)$ through the Lyapunov equation based feedback design [20], where $P(\gamma)$ is the unique positive definite solution to the following parametric algebraic Riccati equation,

$$\begin{aligned} &A^{\mathrm{T}} P(\gamma) + P(\gamma) A - P(\gamma) B B^{\mathrm{T}} P(\gamma) = -\gamma P(\gamma), \\ &\quad \gamma > -2 \min\{\mathrm{Re}(\lambda(A))\}. \end{aligned} \tag{4}$$

The feedback gain matrix $F(\gamma)$ is delay dependent since both B and $P(\gamma)$ contain the information of vector of delays $\vec{\tau} = [\tau_0, \tau_1, \ldots, \tau_N]^{\mathrm{T}}$. To improve the adaptation of the TPF law to