Systems & Control Letters 73 (2014) 6–16

Contents lists available at ScienceDirect

Systems & Control Letters

journal homepage: www.elsevier.com/locate/sysconle

Modeling and estimation for networked systems with multiple random transmission delays and packet losses

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ARTICLE INFO

Article history:
Received 13 June 2013
Received in revised form
28 May 2014
Accepted 25 July 2014

Keywords:
Optimal linear filter
Random transmission delay
Packet loss
Steady state
Orthogonal projection approach

ABSTRACT

In networked systems, data packets are transmitted through networks from a sensor to a data processing center. Due to the unreliability of communication channels, a packet may be delayed even lost during the transmission. At each moment, the data processing center may receive one or multiple data packets or nothing at all. A novel model is developed to describe the possible multiple random transmission delays and data packet losses by employing a group of Bernoulli distributed random variables. It is transformed to a measurement model with multiple random delayed states and noises. Based on the model, an optimal linear filter in the linear minimum variance sense is proposed by using the orthogonal projection approach which is a universal tool to find the optimal linear estimate. It does not have a steady-state performance since it depends on the values of random variables that depict the phenomena of delays and losses at each moment. So it needs to be computed online. To reduce the online computational cost, a suboptimal linear filter dependent on the probabilities of random variables is also proposed. However, it is worth noting that it is linearly optimal among all the linear filters dependent on the probabilities. It can be computed offline since it has the steady-state performance. A sufficient condition of existence for the steady-state performance is given. A simulation example shows the effectiveness.

1. Introduction

In recent years, the networked systems have attracted a lot of attention due to the wide applications in traffic, manufacturing plants, remote processing, and so on [1,2]. In networked systems, random delays and packet losses are unavoidable during the data transmission due to the limited communication capacity [3]. Thus, the research on control and estimation problems over networks is very significant and challenging [4–6].

In networked systems, the phenomena of random delays and packet losses can be usually described by stochastic parameters [7]. For systems with a one-step random delay, a full-order Kalman-like filter has been presented by the method of completing the square [8]. In Ref. [9], two kinds of filters dependent on time stamps and probabilities are respectively designed for multiple time-delay systems by a reorganization innovation approach. For systems with packet losses, some results have been reported, including the robust filter and optimal H-infinity filter based on a linear matrix inequality method [10–12], optimal linear estimators in the linear minimum variance sense based on an innovation analysis approach [13–15], and the corresponding mean square stability analysis [16,17]. Recently, information fusion estimation problems are also studied for multi-sensor systems subject to packet losses [18,19]. However, the results above separately focus on random delays or packet losses, but do not take them together into account.

For systems with both random delays and packet losses, some estimators have been presented [20–23]. In Ref. [20], a time-varying estimator with a finite memory buffer is designed and the upper and lower bounds of the performance are given. In Ref. [21], a packet is sent several times to avoid loss as much as possible, which, however, can bring the network congestion. An H-infinity filter [22] and a fault detection filter [23] are designed by the linear matrix inequality method, respectively. For systems with mixed uncertainties of one-step random delays, packet losses and missing measurements, optimal, suboptimal, and adaptive estimators are respectively designed in Refs. [24–26]. Information fusion estimators are also investigated for multi-sensor systems with random delays and packet losses in Refs. [27–30]. In the new recent literature [31], an optimal linear filter dependent on probabilities in the linear minimum variance sense is designed for systems with random delays and packet losses based on the innovation analysis approach, where a packet is only sent once

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http://dx.doi.org/10.1016/j.sysconle.2014.07.005
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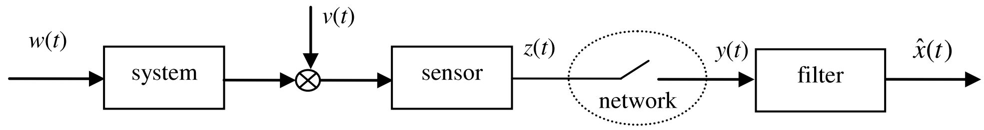

Fig. 1. Sketch of networked systems.

and received on time or lag at most once, or lost. So one packet at most is only used for estimation update at each moment. However, in practice, multiple packets may arrive at the data processing center at the same time as there are possible multiple random delays. This phenomenon usually exists in networked systems such as out-of-sequence measurements [32,33]. Then it is an intuitive idea that the estimation performance will be improved if multiple packets, and not one packet, are used for estimation update at each moment. This motivates the work of this paper.

In this paper, to avoid the network congestion, a data packet of the sensor is only sent once to the data processing center at each moment through networks (see Fig. 1). Due to random delays and packet losses, possibly one or multiple data packets, or nothing arrives at the data processing center at each moment. This is a more general case in networked systems, including out of sequence measurements as a special case. A novel model is developed to depict these phenomena by employing a group of Bernoulli-distributed random variables. Here, we assume that the transmitted data are with time stamps, which can be available in most networks. Differently from Ref. [21] where a packet is sent multiple times, here one packet is only sent once to avoid the network congestion. Therefore, one packet can be only received once at most. Also differently from the recent literature [31] where one packet at most is received and used for estimation update by the filter at each moment, here multiple packets may arrive at the data processing center at each moment and all will be used for estimation update. The developed model is transformed to the measurement model with multiple random delayed states and noises. Using the orthogonal projection approach that is a universal tool to find the optimal linear estimate [34], a non-augmented optimal linear full-order filter is proposed in the linear minimum variance sense. However, it does not have the steady-state performance as it depends on the values of random variables that describe the phenomena of random delays and packet losses at each moment. Therefore, it needs to be computed in real time. To reduce the online computational burden, a suboptimal linear filter dependent on the probabilities of random variables is also proposed. Its advantage is that it has the steady-state performance. It can reduce online computational cost as it can be implemented offline. A sufficient condition of existence for the steady-state filter is given. Two kinds of filters are compared on the accuracy and the online computational cost. The main contributions of this paper include the following: (a) A novel model that describes multiple random transmission delays and packet losses is developed. (b) A full-order optimal linear filter dependent on the values of random variables that depict the phenomena of delays and losses is presented. (c) A suboptimal linear filter dependent on the probabilities of random variables is presented.

2. Problem formulation

Consider a discrete-time linear stochastic system:

$$x(t+1) = \Phi x(t) + \Gamma w(t) \tag{1}$$

$$z(t) = Hx(t) + v(t) \tag{2}$$

where $x(t) \in R^n$ is the state, $z(t) \in R^m$ is the measured output to be sent to the data processing center/filter through networks, $w(t) \in R^r$ and $v(t) \in R^m$ are, respectively, the process noise and measurement noise, and Φ, Γ and H are constant matrices with suitable dimensions. For the brevity of notations, a time-invariant system is only taken into account. However, the results obtained later can be easily extended to a linear time-varying system.

Assume that the sampling and sending rates of the sensor and the receiving rate of the filter are synchronous and clock driven. There exist the bounded d-step transmission delays and possible consecutive packet losses during the data transmission from the sensor to the filter through the network. Moreover, any packet whose transmission delays are more than d steps is considered as being lost. To avoid the network congestion, a packet at sensor side is only sent once at each moment. Due to random delays and losses, one or multiple packets or no data arrive at the data processing center/filter at each moment, which can be met in the case of multiple radios at the filter side. Then, the following model for the measurements received by the filter is adopted:

$$y(t) = \begin{bmatrix} \xi_0(t)z(t) \\ (1-\xi_0(t-1))\xi_1(t)z(t-1) \\ (1-\xi_0(t-2))(1-\xi_1(t-1))\xi_2(t)z(t-2) \\ \vdots \\ \prod_{i=0}^{d-1}(1-\xi_i(t-d+i))\xi_d(t)z(t-d) \end{bmatrix} \tag{3}$$

where $\xi_i(t),\ i = 0, 1, \ldots, d$ are mutually uncorrelated Bernoulli random variables with the known probabilities $\text{Prob}\{\xi_i(t) = 1\} = \alpha_i$ and $\text{Prob}\{\xi_i(t) = 0\} = 1 - \alpha_i$ with $0 \le \alpha_i \le 1$, and are uncorrelated with other random variables.

Model (3) describes the bounded transmission delays and multiple packet losses in networked systems, where the packet losses can be consecutive. Taking $d = 2$ as an example, model (3) describes the following several cases of data arrivals at moment t:

(a) Only $z(t)$ arrives if $\xi_0(t) = 1$, $(1-\xi_0(t-1))\xi_1(t) = 0$ (i.e., $\xi_1(t) = 0$ or $\xi_0(t-1) = 1$), and $(1-\xi_0(t-2))(1-\xi_1(t-1))\xi_2(t) = 0$ (i.e., $\xi_2(t) = 0$ or $\xi_1(t-1) = 1$ or $\xi_0(t-2) = 1$);
(b) Only $z(t-1)$ arrives if $\xi_0(t) = 0$, $(1-\xi_0(t-1))\xi_1(t) = 1$ (i.e., $\xi_1(t) = 1$ and $\xi_0(t-1) = 0$), and $(1-\xi_0(t-2))(1-\xi_1(t-1))\xi_2(t) = 0$ (i.e., $\xi_2(t) = 0$ or $\xi_1(t-1) = 1$ or $\xi_0(t-2) = 1$);
(c) Only $z(t-2)$ arrives if $\xi_0(t) = 0$, $(1-\xi_0(t-1))\xi_1(t) = 0$ (i.e., $\xi_1(t) = 0$ or $\xi_0(t-1) = 1$), and $(1-\xi_0(t-2))(1-\xi_1(t-1))\xi_2(t) = 1$ (i.e., $\xi_2(t) = 1$ and $\xi_1(t-1) = 0$ and $\xi_0(t-2) = 0$);
(d) Both $z(t)$ and $z(t-1)$ arrive if $\xi_0(t) = 1$, $(1-\xi_0(t-1))\xi_1(t) = 1$ (i.e., $\xi_1(t) = 1$ and $\xi_0(t-1) = 0$), and $(1-\xi_0(t-2))(1-\xi_1(t-1))\xi_2(t) = 0$ (i.e., $\xi_2(t) = 0$ or $\xi_1(t-1) = 1$ or $\xi_0(t-2) = 1$);
(e) Both $z(t)$ and $z(t-2)$ arrive if $\xi_0(t) = 1$, $(1-\xi_0(t-1))\xi_1(t) = 0$ (i.e., $\xi_1(t) = 0$ or $\xi_0(t-1) = 1$), and $(1-\xi_0(t-2))(1-\xi_1(t-1))\xi_2(t) = 1$ (i.e., $\xi_2(t) = 1$ and $\xi_1(t-1) = 0$ and $\xi_0(t-2) = 0$);
(f) Both $z(t-1)$ and $z(t-2)$ arrive if $\xi_0(t) = 0$, $(1-\xi_0(t-1))\xi_1(t) = 1$ (i.e., $\xi_1(t) = 1$ and $\xi_0(t-1) = 0$), and $(1-\xi_0(t-2))(1-\xi_1(t-1))\xi_2(t) = 1$ (i.e., $\xi_2(t) = 1$ and $\xi_1(t-1) = 0$ and $\xi_0(t-2) = 0$);
(g) $z(t)$, $z(t-1)$, and $z(t-2)$ all arrive if $\xi_0(t) = 1$, $(1-\xi_0(t-1))\xi_1(t) = 1$ (i.e., $\xi_1(t) = 1$ and $\xi_0(t-1) = 0$), and $(1-\xi_0(t-2))(1-\xi_1(t-1))\xi_2(t) = 1$ (i.e., $\xi_2(t) = 1$ and $\xi_1(t-1) = 0$ and $\xi_0(t-2) = 0$);