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A computational approach to synthesizing guards for hybrid systems

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ABSTRACT

We propose a technique for synthesizing switching guards for hybrid systems to satisfy a given state-based safety constraint. Using techniques from sum of squares (SOS) optimization, we design guards defined by semialgebraic sets that trigger mode switches, and we guarantee that the synthesized switching policy does not allow Zeno executions. We demonstrate our approach on an example of switched affine systems and on an application to traffic ramp metering.

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1. Introduction

Hybrid systems have emerged as a powerful modeling paradigm for complex systems comprised of continuous and discrete components. Often, the discrete mode at any given time can be chosen by a controller. Examples of such systems include traffic networks where the vehicle flow rate is modeled as a continuous-valued variable whose evolution is governed by discrete choices of intersection signals and ramp metering devices. An important task for such systems is to design a policy for switching among the modes to satisfy a *safety* property whereby the system is guaranteed not to enter an unsafe region of the state space (*e.g.* to maintain a certain traffic throughput or to prevent queues from growing too large).

Verifying safety properties and synthesizing safe control strategies for hybrid systems have received considerable attention; *e.g.*, [1–8]. One common approach to the problem of controller synthesis is to calculate a controlled invariant set via an iterative algorithm [9]. The algorithm is initialized with the safe set and iteratively removes trajectories that may be forced to exit the set due to disturbance inputs or system dynamics, thereby eliminating choices for the discrete mode at each time. If the algorithm terminates at a fixed point, this final set is the maximal controlled invariant set, and a least restrictive controller is obtained as a byproduct of the computation [10]. Each step of the iteration requires computing *controllable* and *uncontrollable predecessors* and then solving a *reach–avoid* problem [9] from these predecessors. These subproblems are often formulated as the solution to a Hamilton–Jacobi (HJ) equation (or a pair of coupled HJ equations) [11,9,12].

The primary difficulty of such methods is that the solution of the HJ equations is, in general, computationally taxing. In addition, solution approaches often suffer from numerical difficulties caused by discontinuities in the Hamiltonian [11]. Finally, nearly all computational approaches, such as the prevalent *level-set method* [2], require numerical approximation whose accuracy must be considered. For example, if the numerical approximation is not contained within the maximal controlled invariant set, the synthesis algorithm may identify unsafe states as safe. For some classes of hybrid systems, solutions to HJ equations are efficiently computable [13]; however the class of such systems is very limited.

In this work, we synthesize switching guards that ensure that the hybrid system satisfies a state-based safety constraint using sum of squares (SOS) programming. We consider hybrid systems with a finite number of modes in which the state evolution is governed by a differential inclusion with no continuous control input, and we synthesize guards that trigger transitions between modes. Guards are assumed to be *semialgebraic* sets, *i.e.* a guard is a subset of the continuous state space which satisfies a collection of polynomial inequalities and equalities. Other applications of SOS programming to control theory include region-of-attraction analysis and Lyapunov function calculation, [14,15], hybrid system verification, [6], and calculation of finite-time invariant regions, [16] (see [17] for an overview).

Our switching guard synthesis procedure relies on knowing the reach set from a given set in a particular mode or at least an overapproximation of this set. Finding such sets can be difficult and is an active area of research. The focus of this paper is on classes of systems where the computation of reach sets is amenable to analytical or numerical procedures, and the difficulty in controller synthesis lies in choosing when to switch between discrete modes. We offer

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a synthesis procedure for this task which relies on SOS programming and demonstrate our approach on several examples.

Section 2 introduces notation and reviews hybrid systems and SOS programming. Section 3 states the problem formulation, and Section 4 presents the guard synthesis approach. In Section 5, we apply this method to onramp metering for freeway traffic control. We offer directions for future research in Section 6. This paper extends our conference paper [18]. Extensions include specializing our results to the case of switched affine systems and full development of an extensible application to freeway traffic control.

2. Preliminaries

2.1. Notation

The set $\mathbb{R}_{\geq 0}$ (resp. $\mathbb{R}_{\leq 0}$) is the set of nonnegative (resp. nonpositive) real numbers. For a set X, 2^X is the set of all subsets of X and $\mathbf{cl}(X)$ is the closure of X. For a vector v, $\mathrm{Dim}(v)$ is the dimension of v. The notation 0_n denotes the n-dimensional vector of zeros, and if the dimension is evident, the subscript is suppressed. We denote elementwise nonnegativity of a vector v by $v \succeq 0$. An asterisk $(*)$ used as a subscript denotes a placeholder to be replaced with elements from an index set which is evident from context.

2.2. Hybrid systems

A *hybrid system* is a tuple $H = (Q, X, I, f, R, \mathcal{G})$ where the total state space $Q \times X$ consists of a finite set Q of *modes* and a continuous state space $X = \mathbb{R}^n$. The system is initialized in a set $I \subseteq Q \times X$, and we define $I(q) \triangleq \{x : (q, x) \in I\}$. We consider differential inclusions such that

$$\dot{x}(t) \in f(q, x(t)) \quad \text{for almost all } t \tag{1}$$

where $f(\cdot, \cdot) : Q \times X \to 2^X$ constrains the continuous evolution while in mode q. Mild assumptions on $f(q, \cdot)$ guarantee the existence and absolute continuity of solutions [19, Section 3.3]. In particular, we further assume that $f(q, \cdot)$ is locally bounded. This formulation is general and can accommodate, for example, parameter uncertainty or disturbance inputs.

We define the reset map as follows: $R(\cdot, \cdot, \cdot) : Q \times Q \times X \to 2^X$ where $R(q, q', x) \subseteq X$ is the set of continuous states which can be reached when the system undergoes a transition from discrete state q to q' while at $x \in X$. We denote the domain of R for fixed q, q' by $\mathcal{R}_{q\to q'} \triangleq \mathrm{Dom}(R(q, q', \cdot)) \subseteq X$. For a set $M \subseteq \mathcal{R}_{q\to q'}$, we understand $R(q, q', M) \triangleq \bigcup_{x\in M} R(q, q', x)$. Note that if a transition from q to q' is not possible, then $\mathcal{R}_{q\to q'} = \emptyset$.

A *set of guards* $\mathcal{G}$ for a hybrid system is a collection of sets $\mathcal{G} = \{\mathcal{G}_{q\to q'}\}_{q,q'\in Q}$ such that

$$\mathcal{G}_{q\to q'} \subseteq \mathcal{R}_{q\to q'}. \tag{2}$$

Each $\mathcal{G}_{q\to q'}$ is called a *guard*, and if $x \in \mathcal{G}_{q\to q'}$, we say the guard from mode q to q' is *active*. Let

$$\mathcal{G}_q \triangleq \bigcup_{q'\in Q} \mathcal{G}_{q\to q'}. \tag{3}$$

The purpose of the guards is to trigger mode transitions and the corresponding reset of the continuous state dictated by the reset map. In this work, we consider synthesizing a set of guards so that the hybrid system satisfies a safety property.

An *execution* of a hybrid system H is a sequence of mode transition times $\{\tau_i\}_{i=1}^N$ with $\tau_0 = 0$, $\tau_i \leq \tau_{i+1}$ along with a state trajectory $(q(t), x(t))$ where $q(t)$ is constant and $x(t) \in X \setminus \mathcal{G}_{q(t)}$ for all $t \in [\tau_i, \tau_{i+1})$ if $\tau_i < \tau_{i+1}$, and $\dot{x}(t) \in f(q(t), x(t))$ for almost all $t \in [\tau_i, \tau_{i+1})$ if $\tau_i < \tau_{i+1}$. We allow the case where $N = \infty$ and the case where $N < \infty$, $\tau_N = \infty$. We denote the continuous state immediately prior to the ith transition by $x(\tau'_{i-1})$, *i.e.* $x(\tau'_{i-1}) \triangleq \lim_{t\to\tau_i^-} x(t)$ if $\tau_{i-1} < \tau_i$, or $x(\tau'_{i-1}) \triangleq x(\tau_i)$ if $\tau_{i-1} = \tau_i$. We further require $x(\tau'_i) \in \mathcal{G}_{q(\tau_i)\to q(\tau_{i+1})}$, and $x(\tau_{i+1}) \in R(q(\tau_i), q(\tau_{i+1}), x(\tau'_i))$ for $i = 1, \ldots, N-2$ and for $i = N-1$ if $q(\tau_N) \neq q(\tau_{N-1})$. If $N = \infty$ but $\sup_i \tau_i < \infty$, the execution is called *Zeno*. For a detailed discussion of the types of executions possible in hybrid systems, see [20].

2.3. Sum of squares programming

For a variable x taking values in $\mathbb{R}^n$, we denote by $\mathbb{R}[x]$ the set of all polynomials in x. Define

$$\Sigma[x] \triangleq \left\{ \sigma(x) \in \mathbb{R}[x] : \sigma(x) = \sum_{i=1}^{m} f_i(x)^2, f_i(x) \in \mathbb{R}[x] \right\}. \tag{4}$$

A polynomial $\sigma(x) \in \Sigma[x]$ is called a *sum of squares (SOS)* polynomial. Given $\{p_i(x)\}_{i=0}^m$ with $p_i \in \mathbb{R}[x]$, the problem of finding $\{q_i(x)\}_{i=1}^m$ with $q_i(x) \in \mathbb{R}[x]$ (or $q_i(x) \in \Sigma[x]$, or a mix of constraints for different i's) such that

$$p_0(x) + \sum_{i=1}^{m} q_i(x) p_i(x) \in \Sigma[x] \tag{5}$$

is a semidefinite program [17], and the MATLAB toolbox SOSTOOLS [21] transforms *SOS programs* of the form (5) into semidefinite programs.

3. Problem formulation

Consider an *unsafe set* $U \subseteq Q \times X$ which includes undesirable regions of the state space. Given a hybrid system H, we call an execution of H *unsafe* if $(q(t), x(t)) \in U$ for some $t \in [0, \tau_N]$. We call H *safe* if there does not exist an unsafe execution of H.

Guard Synthesis Problem. Given a hybrid system H with unspecified guards and an unsafe set $U \subseteq Q \times X$, synthesize a set of guards $\mathcal{G} = \{\mathcal{G}_{q\to q'}\}_{q,q'\in Q}$ such that H is safe.

For $S \subset X$, we call $\Phi \subset X$ an *overapproximation of the reach set* from (q, S) if Φ contains all trajectories of the continuous dynamics in mode q that originate in S until a guard is encountered. Specifically, Φ is an overapproximation of the reach set from (q, S) if for all $T > 0$

$$\left.\begin{array}{l} x(0) \in S \\ \dot{x}(t) \in f(q, x(t)) \quad \text{for almost all } t \in [0, T) \\ x(t) \in (X \setminus \mathcal{G}_q) \quad \text{for all } t \in [0, T) \end{array}\right\}$$
$$\text{implies} \begin{array}{l} x(t) \in \Phi \quad \forall t \in [0, T) \text{ and} \\ \lim_{t\to T^-} x(t) \in \Phi. \end{array} \tag{6}$$

As we only concern ourselves with overapproximations of reach sets in this work, we will often refer to such overapproximations as simply *reach sets*. We define the set-valued function $\textsc{Reach}(\cdot, \cdot)$ as follows:

$$\textsc{Reach}(q, S) \triangleq \{\Phi : \Phi \text{ is a reach set from } (q, S)\}. \tag{7}$$

Note that if $S \subset X$ is a positively invariant set for the dynamics $\dot{x} \in f(q, x)$, then $S \in \textsc{Reach}(q, S)$.

A number of techniques exist for obtaining such overapproximations. For example, in [6], the authors consider scalar-valued "barrier functions" $B_q(x)$ and use the fact that if

$$\nabla B_q(x)^T v \geq 0 \quad \text{for all } v \in f(q, x), \text{ for all } x \in (X \setminus \mathcal{G}_q)$$
$$\text{s.t. } B_q(x) = 0 \tag{8}$$

then $\{x : B_q(x) \geq 0\} \in \textsc{Reach}(q, \{x : B_q(x) \geq 0\})$. The authors of [6] propose a technique for constructing such barrier functions

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