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Adaptive disturbance attenuation via logic-based switching

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ABSTRACT

The problem of attenuating unknown and possibly time-varying disturbances acting on a linear time-invariant dynamical system is addressed by means of an adaptive switching control approach. Given a family of pre-designed stabilizing controllers, a supervisory unit infers in real-time the potential behavior of each candidate controller and selects the one providing the best potential performance. To this aim, a set of test functionals is defined, which is shown to enjoy favorable inference properties under certain assumptions on the nature of the disturbances. Both persistent-memory and finite-memory test functionals are analyzed. Further, an implementation of the switching controller is proposed which always guarantees stability of the feedback loop, even if the disturbance characteristics are such that the switching is persistent. Simulation results are provided to show the effectiveness of the proposed method.

1. Introduction

One of the most relevant problems in control design is that of reducing the effects of undesirable input signals on the output of the process to be controlled. This issue becomes even more challenging when the disturbance characteristics are unknown and possibly time-varying. Much interest to adaptive solutions for the problem of noise attenuation has been raised with respect to narrow-band (sinusoidal and periodic) disturbances, and the proposed methods involve in general either direct or indirect approaches. Recent contributions can be found in [1–6]. The indirect scheme is composed of two steps, that is, disturbance frequency estimation and employment of the estimated frequency in a disturbance cancellation architecture for known frequency. On the other hand, in a direct approach the controller synthesis algorithms are inherently designed to attenuate the disturbances. Disturbance rejection/attenuation arises in several diversified contexts such as: active noise control [7]; noise cancellation in an acoustic duct [8]; vibration attenuation for helicopter rotors [9]; biomass productivity in a fed-batch reactor [10]; disturbance torque compensation in electric machines [11]; eccentricity compensation [12]; disk drive control [13]; disturbance attenuation for Adaptive Optics applications for ground-based telescopes [14]. Notice that, while many of the former examples are in turn focused on narrow-band disturbance applications, in other contexts, e.g., in the adaptive optics case, the disturbance frequency profiles are inherently broadband.

In this paper, we address the problem of the attenuation of disturbances with unknown and possibly time-varying characteristics by means of the *Adaptive Switching Control* (ASC) approach, which has been proposed as an alternative solution to the classical *Adaptive control* paradigm (see [15–20]). As well known, the ASC approach consists of two independent steps: the synthesis of a finite family of controllers (each one suitable for a specific operating condition), which can be performed according to any design technique; and the definition of a switching rule. An extensive discussion on the possible advantages of the switching control approach compared to the classical adaptive one can be found for example in [21]. With respect to the problem of interest, we can mention that the ASC paradigm, thanks to its modularity, is well suited to being applied in all the above-mentioned contexts, since the specific disturbance model can be accounted for during the controller design phase. Specifically, the solution that we propose in this paper is based on the assumptions that: the plant model is linear time-invariant and described by a known transfer function; all the pre-synthesized controllers stabilize the plant; and, for any possible operating conditions, at least one of the controllers is able to

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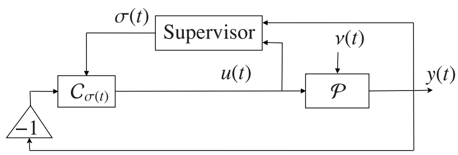

Fig. 1. Overall control scheme.

achieve a certain prescribed performance level. At each time instant, the controller providing the best potential performance with respect to the current operating condition is selected according to a *Hysteresis Switching Logic* (HSL), see [15]; the potential performance of each controller is quantified in terms of test functionals defined on the basis of the plant input/output data.

The paper is organized as follows. Section 2 defines the problem of interest and gives insight into the controller selection criterion. In Section 3, the properties of the test functionals are studied under certain assumptions on the nature of the disturbances. Section 4 shows that stability of the switched system can be ensured by adopting a particular implementation of the switched controller. The results obtained within several simulation settings are presented in Section 5, and some concluding remarks are provided in Section 6. The Proofs of the main results are provided in the Appendix.

2. Problem setting

Consider a single-input single-output (SISO) linear time-invariant (LTI) dynamical system whose input–output behavior can be described by the difference equation

$$A(d)y(t) = B(d)u(t) + \nu(t), \tag{1}$$

where $t \in \mathbb{Z}_+$, $\mathbb{Z}_+ := \{0, 1, \ldots\}$, denote discrete time instants, $y(t)$ is the system output, $u(t)$ the control input, and $\nu(t)$ an unknown bounded disturbance acting on the system. The polynomials $A(d) = 1 + a_1 d + \cdots + a_{n_a} d^{n_a}$ and $B(d) = b_1 d + \cdots + b_{n_b} d^{n_b}$ in the unit backward shift operator d are known and have strictly Schur greatest common divisor (g.c.d.).

Remark 1. We underline that the model provided in (1) can be used without loss of generality. In fact, a general LTI model of the form

$$\overline{A}(d)y(t) = \frac{B(d)}{F(d)}u(t) + \frac{H(d)}{R(d)}e(t), \tag{2}$$

where $e(t)$ is a zero mean white noise with variance σ_e^2, and $B(d)$ and $F(d)$ are coprime polynomials, can be written as in (1) by defining $A(d) := \overline{A}(d)F(d)$ and $\nu(t) := (H(d)F(d)/R(d))\, e(t)$. In the simulation setup presented in Section 5, for example, we will resort to the model in (1) with the disturbance $\nu(t)$ obtained as the output of a suitable filter.

We denote by $\mathcal{P}$ the plant in (1) having transfer function $P(d) = B(d)/A(d)$. The problem of interest is that of attenuating the disturbance $\nu(t)$ by regulating the plant output $y(t)$ around zero. We suppose that a non-negligible uncertainty affects the a priori available disturbance model, so that a single robust LTI controller cannot achieve satisfactory performance within the whole uncertainty set.

The solution that we propose for the problem of disturbance attenuation relies on the ASC paradigm; in fact, the control architecture is composed of a finite family of pre-designed controllers supervised by a high-level switching logic. The controllers are supposed to have been synthesized off-line, according to any design technique, so that: each of the pre-synthesized controllers stabilize the plant; and, for any possible operating conditions, at least one of the controllers is able to achieve a certain prescribed performance level (for example, in terms of disturbance-to-output energy gain). Then, at each time instant, the supervisory unit infers the potential behavior of each candidate controller and selects the one providing the best potential performance, which is quantified in terms of test functionals defined on the basis of the plant input/output data.

Let $\mathscr{C} := \{\mathcal{C}_i,\ i \in \overleftarrow{N}\}$ denote the family of pre-designed candidate controllers, where $\overleftarrow{N} := \{1, 2, \ldots, N\}$. The transfer function of the ith controller is $C_i(d) = S_i(d)/R_i(d)$ with the polynomials $R_i(d) = 1 + r_{i1}d + \cdots + r_{in_{r_i}} d^{n_{r_i}}$ and $S_i(d) = s_{i0} + s_{i1}d + \cdots + s_{in_{s_i}} d^{n_{s_i}}$ having strictly Schur g.c.d. At each time instant, the selected controller belonging to $\mathscr{C}$ is identified by means of the switching signal $\sigma(\cdot) : \mathbb{Z}_+ \to \overleftarrow{N}$. Accordingly, we denote by $\mathcal{C}_{\sigma(t)}$ the switching controller (or multi-controller), with the understanding that, on all the time intervals on which $\sigma(t)$ is constant and equal to a certain i, the multi-controller takes the form of a LTI system having transfer function equal to $C_i(d)$. We defer any discussion on the internal structure of the multi-controller to Section 4, where it will be shown how a suitable implementation of such a block always preserves stability under arbitrary switching. The overall control scheme is depicted in Fig. 1.

2.1. Controller selection

In this section, a criterion is proposed for selecting, among the controllers belonging to $\mathscr{C}$, the one to be put in feedback to the plant. To this end, at any time t, a set of test functionals $\Pi(t) := \left\{\Pi_i(t),\ i \in \overleftarrow{N}\right\}$ is computed, each one quantifying the performance achievable by the use of the related candidate controller $\mathcal{C}_i$. Each test functional is computed only on the grounds of the plant input/output data without the necessity of inserting the corresponding controller in the feedback loop.

Let

$$\varepsilon(t) := A(d)\, y(t) - B(d)\, u(t) \tag{3}$$

denote the prediction error, where $y(t)$ and $u(t)$ are the output and input data produced by the switching system $(\mathcal{P}/\mathcal{C}_{\sigma(t)})$ composed of the feedback interconnection of $\mathcal{P}$ and $\mathcal{C}_{\sigma(t)}$. We underline that, for any $t \geq n = \max\{n_a, n_b\}$, the prediction error $\varepsilon(t)$ coincides with the disturbance $\nu(t)$. Hence, the potential performance achievable by a certain candidate controller $\mathcal{C}_i$ can be evaluated by filtering the prediction error $\varepsilon(t)$ with a suitable transfer matrix $\Sigma_i(d)$ related to the potential loop $(\mathcal{P}/\mathcal{C}_i)$, defined as the feedback interconnection of the plant $\mathcal{P}$ with the controller $\mathcal{C}_i$.

In particular, we consider the weighted mixed-sensitivity $\Sigma_i(d)$ related to the loop $(\mathcal{P}/\mathcal{C}_i)$ and defined as

$$\Sigma_i(d) := \frac{1}{\chi_i(d)}\, L_i(d) \tag{4}$$

where

$$L_i(d) = \begin{bmatrix} R_i(d) \\ -\eta\, S_i(d) \end{bmatrix}$$

and $\chi_i(d) = A(d)\, R_i(d) + B(d)\, S_i(d)$ is the characteristic polynomial of $(\mathcal{P}/\mathcal{C}_i)$. It can be seen that the two elements of $\Sigma_i(d)$ represent the disturbance-to-output and, respectively, disturbance-to-control transfer functions of $(\mathcal{P}/\mathcal{C}_i)$. The latter one is weighted by a nonnegative scalar η which can be tuned to give more or less importance to the contribution of the control input in the performance evaluation.

Then in order to compute, for each $i \in \overleftarrow{N}$, the hypothetical weighted response

$$z_i = [y_i\ \eta\, u_i]^\top \tag{5}$$