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Simultaneous compensation of input and state delays for nonlinear systems

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ABSTRACT

The problem of compensation of arbitrary large input delay for nonlinear systems was solved recently with the introduction of the nonlinear predictor feedback. In this paper we solve the problem of compensation of input delay for nonlinear systems with simultaneous input and state delays of arbitrary length. The key challenge, in contrast to the case of only input delay, is that the input delay-free system (on which the design and stability proof of the closed-loop system under predictor feedback are based) is infinite-dimensional. We resolve this challenge and we design the predictor feedback law that compensates the input delay. We prove global asymptotic stability of the closed-loop system using two different techniques—one based on the construction of a Lyapunov functional, and one using estimates on solutions. We present two examples, one of a nonlinear delay system in the feedforward form with input delay, and one of a scalar, linear system with simultaneous input and state delays.

1. Introduction

Nonlinear delay systems are ubiquitous in applications. A non-exhaustive list includes traffic systems [1], additive manufacturing [2], oil drilling [3], automotive engines [4] and catalysts [5,6], energy systems, such as, for example, cooling systems [7], and networked control systems [8].

Nonlinear systems with state delays represent an advanced research area [9–15]. Numerous results also exist on the control and analysis of nonlinear systems with input delays [16–27]. Few results exist on the compensation of input delay for systems with simultaneous delay on the state, even for linear systems [28–30]. In [28] and [29] predictor feedback designs are developed, exploiting the special structure of the linear systems under consideration (systems in the feedback and feedforward form respectively), and in [30] a predictor feedback design is presented for general linear systems. Even fewer are the results dealing with the analysis and control of nonlinear systems with simultaneous input and state delays [31]. In [32] a predictor feedback law is developed for the compensation of input delay for a special class of nonlinear delay systems, namely, systems in the strict-feedback form with a state delay on the virtual input.

We consider nonlinear systems with simultaneous long discrete input delay and long (potentially distributed) state delay (the problem of the compensation of a distributed input delay is a different problem that goes beyond the predictor feedback approach that we present here, and therefore, we do not consider this problem in the present paper). We design a nonlinear predictor feedback law which employs, in a nominal feedback law that stabilizes the system with only the state delay, the predictor of the state over a prediction horizon equal to the length of the input delay, and hence, it achieves compensation of the input delay (Section 2). (Predictor feedback designs that also achieve compensation of the state delay, by exploiting the special structure of the system under consideration, are presented in [28] and in [32] for systems in the strict-feedback form with delays on the virtual inputs.) For nonlinear delay systems that are forward complete in the absence of the input delay we prove global asymptotic stability of the closed-loop system with the aid of a Lyapunov functional that we construct, based on the introduction of an infinite-dimensional backstepping transformation of the actuator state (Section 3).

We also present an alternative proof of global asymptotic stability by constructing estimates on the solutions of the closed-loop system and exploiting the facts that the input delay is compensated after a finite time-interval and that the system in the absence of only the input delay is forward complete (Section 4). We present a simulation example of a second-order nonlinear system in the strict-feedforward form with both input and state delays

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(Section 5). We also illustrate the linear case through an example of a scalar system with simultaneous input and state delays (Section 5).

Notation. We use the common definition of class $\mathcal{K}$, $\mathcal{K}_\infty$ and $\mathcal{KL}$ functions from [33]. For an n-vector, the norm $|\cdot|$ denotes the usual Euclidean norm. We denote by $C^j(A;\Omega)$ the class of functions, taking values in Ω, that have continuous derivatives of order j in A. We denote by $L^\infty(A;\Omega)$ the space of measurable and bounded functions defined on A and taking values in Ω. For a given $D_1 \geq 0$ and a function $\phi \in L^\infty([-D_1,0];\mathbb{R}^n)$ we denote by $\|\phi\|_{D_1}$ its supremum over $[-D_1,0]$, i.e., $\|\phi\|_{D_1} = \sup_{s\in[-D_1,0]}|\phi(s)|$. For a function $X : [-D_1,\infty) \to \mathbb{R}^n$, for all $t \geq 0$, the function X_t is defined by $X_t(s) = X(t+s)$, for all $s \in [-D_1,0]$. For a function $U : [-D_2,\infty) \to \mathbb{R}^n$, for all $t \geq 0$, the function U_t is defined by $U_t(s) = U(t+s)$, for all $s \in [-D_2,0]$. For a function $P : [-D_1-D_2,\infty) \to \mathbb{R}^n$, for all $\theta \geq -D_2$, the function P_θ is defined by $P_\theta(s) = P(\theta+s)$, for all $s \in [-D_1,0]$. Any relation in which the time t appears holds for all $t \geq 0$, unless stated otherwise.

2. Problem formulation and controller design

We consider the following system

$$\dot{X}(t) = f\left(X_t, U(t-D_2)\right), \tag{1}$$

for $t \geq 0$, where $f : C([-D_1,0];\mathbb{R}^n) \times \mathbb{R} \to \mathbb{R}^n$ is a locally Lipschitz mapping with $f(0,0) = 0$, and $D_1, D_2 \geq 0$. For designing a stabilizing feedback law for (1) one needs two ingredients. First, one needs a nominal feedback law that stabilizes system (1) when there is no input delay, i.e., system

$$\dot{X}(t) = f\left(X_t, U(t)\right). \tag{2}$$

The second ingredient one needs is the D_2-time units ahead predictor of X, that is, the signal P that satisfies $P(s) = X(s+D_2)$, for all $s \geq -D_1-D_2$. The controller that stabilizes system (1) and compensates the input delay is then given for $t \geq 0$ by

$$U(t) = \kappa\left(P_t\right), \tag{3}$$

where

$$P(\theta) = X(t) + \int_{t-D_2}^{\theta} f\left(P_s, U(s)\right) ds, \quad \text{for all } t-D_2 \leq \theta \leq t, \tag{4}$$

with initial condition given by

$$P(s) = X\left(s+D_2\right), \quad \text{for all } -D_1-D_2 \leq s \leq -D_2 \tag{5}$$

$$P(\theta) = X(0) + \int_{-D_2}^{\theta} f\left(P_\sigma, U(\sigma)\right) d\sigma, \quad \text{for all } -D_2 \leq \theta \leq 0. \tag{6}$$

The fact that P is the D_2-time units ahead predictor of X can be seen as follows. Performing the change of variables $t = \theta + D_2$, for all $t-D_2 \leq \theta \leq t$ in (1) and integrating starting at $\theta = t-D_2$ we get that

$$X\left(\theta+D_2\right) = X(t) + \int_{t-D_2}^{\theta} f\left(X_{s+D_2}, U(s)\right) ds. \tag{7}$$

Defining $P(\theta) = X(\theta+D_2)$, for all $t-D_2 \leq \theta \leq t$ and using the fact that P satisfies (5), we conclude that the signal P defined by (4), with initial conditions (5), (6) satisfies $P(s) = X(s+D_2)$, for all $s \geq -D_1-D_2$.

3. Lyapunov-based stability analysis

Assumption 1. System (2) is forward complete.

Assumption 1 guarantees that for every initial condition $X_0 \in C([-D_1,0];\mathbb{R}^n)$ and for every locally bounded input signal U the corresponding solution is defined for all $t \geq 0$.

Assumption 2. There exist a locally Lipschitz feedback law $\kappa : C([-D_1,0];\mathbb{R}^n) \to \mathbb{R}$ with $\kappa(0) = 0$ and a class $\mathcal{K}_\infty$ function α such that for all $\phi \in C([-D_1,0];\mathbb{R}^n)$

$$|\kappa(\phi)| \leq \alpha\left(\|\phi\|_{D_1}\right), \tag{8}$$

a locally Lipschitz functional $S : C([-D_1,0];\mathbb{R}^n) \to \mathbb{R}_+$, and class $\mathcal{K}_\infty$ functions $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ such that for all $\phi \in C([-D_1,0];\mathbb{R}^n)$ it holds that

$$\alpha_1\left(|\phi(0)|\right) \leq S(\phi) \leq \alpha_2\left(\|\phi\|_{D_1}\right), \tag{9}$$

and along the trajectories of the closed-loop system $\dot{X}(t) = f\left(X_t, \kappa\left(X_t\right) + \omega(t)\right)$, S is continuously differentiable and satisfies for all $\omega \in C([0,+\infty);\mathbb{R})$

$$\dot{S}(t) \leq -\alpha_3\left(S\left(X_t\right)\right) + \alpha_4\left(|\omega(t)|\right), \tag{10}$$

for all $t \geq 0$.

Theorem 1. *Consider system (1) together with the control law (3)–(6). Under Assumptions 1 and 2 there exists a class $\mathcal{KL}$ function β such that for all initial conditions $X_0 \in C([-D_1,0];\mathbb{R}^n)$ and $U_0 \in C([-D_2,0];\mathbb{R})$, that are compatible with the feedback law, that is, they satisfy $U_0(0) = \kappa(P_0)$, there exists a unique solution to the closed-loop system with $X \in C^1([0,+\infty);\mathbb{R}^n)$, $U \in C([0,+\infty);\mathbb{R})$, and the following holds*

$$\sup_{t-D_1\leq\tau\leq t}|X(\tau)| + \sup_{t-D_2\leq\theta\leq t}|U(\theta)| \leq \beta\left(\sup_{-D_1\leq\tau\leq 0}|X(\tau)| + \sup_{-D_2\leq\theta\leq 0}|U(\theta)|, t\right), \tag{11}$$

for all $t \geq 0$.

The proof of Theorem 1 is based on a series of technical lemmas that are presented next.

Lemma 1. *The infinite-dimensional backstepping transformation of the actuator state defined by*

$$W(\theta) = U(\theta) - \kappa\left(P_\theta\right), \quad t-D_2 \leq \theta \leq t, \tag{12}$$

together with the predictor feedback law (3)–(6) transform the system (1) to the "target system" given by

$$\dot{X}(t) = f\left(X_t, \kappa\left(X_t\right) + W\left(t-D_2\right)\right) \tag{13}$$

$$W(t) = 0, \quad \forall t \geq 0. \tag{14}$$

Proof. Using (1) and the fact that $P_{t-D_2} = X_t$ we get (13). With (3) we get (14).

Lemma 2. *The inverse of the infinite-dimensional backstepping transformation defined in (12) is given by*

$$U(\theta) = W(\theta) + \kappa\left(\Pi_\theta\right), \quad t-D_2 \leq \theta \leq t, \tag{15}$$

where

$$\Pi(\theta) = X(t) + \int_{t-D_2}^{\theta} f\left(\Pi_s, \kappa\left(\Pi_s\right) + W(s)\right) ds, \quad \text{for all } t-D_2 \leq \theta \leq t, \tag{16}$$