Applied Acoustics 140 (2018) 39–47

Contents lists available at ScienceDirect

Applied Acoustics

journal homepage: www.elsevier.com/locate/apacoust

Operational transfer path analysis of a piano

Jin Jack Tan[a,b,*,1], Antoine Chaigne[a,2], Antonio Acri[c,d,3]

[a] IWK, University of Music and Performing Arts Vienna, Austria
[b] IMSIA, ENSTA-ParisTech-CNRS-EDF-CEA, France
[c] Politecnico di Milano, Italy
[d] Virtual Vehicle, Austria

ARTICLE INFO

Keywords:
Piano acoustics
Structural vibration
Source identification
Operational transfer path analysis

ABSTRACT

The piano sound is made audible by the vibration of its soundboard. A pianist pushes the key to release a hammer that strikes the strings, which transfer the energy to the soundboard, set it into vibration and the piano sound is heard due to the compression of air surrounding the soundboard. However, as piano is being played, other components such as the rims, cast-iron frame and the lid are also vibrating. This raises a question of how much of their vibrations are contributing to the sound as compared to the soundboard. To answer this question, operational transfer path analysis, a noise source identification technique used widely in automotive acoustics, is carried out on a Bösendorfer 280VC-9 grand piano. The "noise" in a piano system would be the piano sound while the "sources" are soundboard and the aforementioned components. For this particular piano, it is found out that the soundboard is the dominant contributor. However, at high frequencies, the lid contributes the most to the piano sound.

1. Introduction

The study of piano acoustics has traditionally been focused on its piano action [1,2], interaction between a piano hammer and string [3,4], string vibration [5–7], soundboard vibration [8–10] and its radiation [11–13]. As computational power becomes cheaper, it is possible to model the piano as a coupled system that involves the string, soundboard and surrounding air [14,15]. However, this raises a question whether the considered system is complete enough to have a realistic reproduction of the piano sound. The importance and role of a soundboard in piano sound production have been extensively studied [16–21] but not for other components. Is there any other components that are contributing to the piano sound production that has not been accounted for? Anecdotally, when a piano is played, vibration can be felt not only on the soundboard but also on the rim, the frame, the lid etc. In a Bösendorfer piano, spruce, a wood usually used for the soundboard by other manufacturers, is used extensively in building the case of the piano. Bösendorfer claims that the use of spruce, especially on the rim of the piano, allows the whole instrument to vibrate and is the reason that gives the unique Bösendorfer sound [22]. Based on the fact that vibration is felt on other parts of the piano and how Bösendorfer uses spruce extensively, it necessitates an investigation if the vibrations of these parts contribute to the production of the sound.

Current work takes inspiration from noise source identification techniques used commonly in automotive acoustics [23–25]. However, in the case of piano, the "noise" is the resulting piano sound and the "sources" to be identified are the piano components to be investigated. These "sources" may emit different "noise" contributions that characterise the resulting "noise", i.e. the piano sound. One technique that can be used to identify the contribution of the piano components to the final sound is the operational transfer path analysis (OTPA) [26]. OTPA computes a transfer function matrix to relate a set of input/source(s) measurements to output/response(s) measurements. In this case, the inputs are the vibration of the components of piano and the output is the resulting piano sound. Initial result of the work was first presented at the International Congress of Acoustics [27] but more thorough analysis has since been conducted with more convincing and confident results obtained. These results are being presented in this paper. In Section 2, the theory of OTPA is presented. The experiment designed for OTPA is then detailed in Section 3 before the results are shown and discussed in Section 4.

* Corresponding author.
E-mail address: tanjinjack@gmail.com (J.J. Tan).
[1] Present affiliation: Faculty of Engineering and the Environment, University of Southampton Malaysia Campus, Iskandar Puteri, Malaysia.
[2] Present address: 13 allée François Jacob, 78530 BUC, France.
[3] Present affiliation: R&D Department, Brugola OEB Industriale spa, Milan, Italy.

https://doi.org/10.1016/j.apacoust.2018.05.008
Received 6 September 2017; Received in revised form 30 April 2018; Accepted 4 May 2018

2. The operational transfer path analysis (OTPA)

Operational transfer path analysis (OTPA) is a signal processing technique that studies the noise source propagation pathways of a system based on its operational data [28]. This is in contrast to classical transfer path analysis (TPA) where the source propagation pathways are established by means of experimental investigations with specific inputs. Indeed, OTPA was developed in wake of the need of a fast and robust alternative to the classical TPA. In both methods, the source propagation pathways are determined by studying the transfer functions between the sources (inputs) and the responses (outputs). In TPA, the relationship between sources (excitation signals) and responses are estimated by frequency response functions and these are meticulously determined by series of experimental excitation of known forces (e.g. by shaker or impact hammer). Where necessary, part of the system is also removed or isolated. In this way, TPA is able to trace the flow of vibro-acoustic energy from a source through a set of known structure- and air-borne pathways, to a given receiver location by studying the frequency response functions. On the other hand, OTPA measures directly the source and response signals when the system is operated and establishes the source propagation pathways based on the experimentally determined transfer functions. OTPA is a response-response model where measurement data are collected and analysed while the system, when operated, provides the excitation. Detailed comparisons between the classical TPA and OTPA can be found in [29–31].

While OTPA may appear to be simpler to use, it is also prone to error if it is not designed and analysed properly. OTPA requires prior knowledge of the system as neglected pathways could not be easily detected. In a multi-component system, cross-coupling between the components could affect the accuracy of an OTPA model. Several techniques, which are detailed in the following Section 2.2, can be employed to mitigate the effects of cross-coupling [28].

2.1. Theory of OTPA

In a linear system, the input $\mathbf{X}$ and output $\mathbf{Y}$ can be related by:

$$\mathbf{Y}(j\omega) = \mathbf{X}(j\omega)\mathbf{H}(j\omega), \tag{1}$$

where:

$\mathbf{Y}(j\omega)$ is the output vector/matrix at the receivers;
$\mathbf{X}(j\omega)$ is the input vector/matrix at the sources;
$\mathbf{H}(j\omega)$ is the operational transfer function matrix, also known as the transmissibility matrix. The dependency of frequencies for all three matrix is as denoted by $(j\omega)$ [28].

The inputs and outputs signals can be the forces, displacements, velocities or pressures of the components in the system. Given that there are m inputs and n outputs with p set of measurements, Eq. (1) can be written in the expanded form:

$$\begin{bmatrix} y_{11} & \cdots & y_{1n} \\ \vdots & \ddots & \vdots \\ y_{p1} & \cdots & y_{pn} \end{bmatrix} = \begin{bmatrix} x_{11} & \cdots & x_{1m} \\ \vdots & \ddots & \vdots \\ x_{p1} & \cdots & x_{pm} \end{bmatrix} \begin{bmatrix} H_{11} & \cdots & H_{1n} \\ \vdots & \ddots & \vdots \\ H_{m1} & \cdots & H_{mn} \end{bmatrix}, \tag{2}$$

where for clarity purposes, the frequency dependency $j\omega$ is dropped. In order to quantify the contributions of the inputs to the outputs, the transfer function matrix needs to be solved. If the input matrix $\mathbf{X}$ is square and invertible, this can be solved by simply multiplying the inverse of $\mathbf{X}$ on both sides:

$$\mathbf{H} = \mathbf{X}^{-1}\mathbf{Y}. \tag{3}$$

However, in most cases, $p \neq m$. Thus, for the system to be solvable, it is required that the number of measurement sets is larger than or equal to the number of inputs, i.e.

$$p \geqslant m, \tag{4}$$

thereby forming an overdetermined system with residual vector μ:

$$\mathbf{XH} + \mu = \mathbf{Y}. \tag{5}$$

The transmissibility matrix $\mathbf{H}$ can then be obtained via the following equation [28]:

$$\mathbf{H} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{Y} = \mathbf{X}^{+}\mathbf{Y}, \tag{6}$$

where the $+$ superscript denotes the Moore-Penrose pseudo-inverse [32].

2.2. Enhanced OTPA with singular value decomposition and principal component analysis

In essence, the basics of OTPA is analogous to the multiple-input multiple-output (MIMO) technique in experimental modal analysis [33]. However, solving the transfer function $\mathbf{H}$ directly is prone to error if the input signals are highly coherent between each other. High coherence is caused by unavoidable cross-talks between the measurement channels as they are sampled simultaneously. To mitigate this error, an enhanced version of OTPA can be employed. Singular value decomposition (SVD) and principal component analysis (PCA) can be carried out [28,26]. There are two main reasons in using SVD. Firstly, it can be used to solve for $\mathbf{X}^{+}$, even though it is not the only way to solve for Moore–Penrose pseudo-inverse. Secondly, the singular value matrix can later be repurposed to carry out PCA to reduce the measurement noise.

The input matrix $\mathbf{X}$, as decomposed by economy size SVD, can be written as

$$\mathbf{X} = \mathbf{U\Sigma V^T}, \tag{7}$$

where

$\mathbf{U}$ is a unitary column-orthogonal matrix;
$\mathbf{\Sigma}$ is a square diagonal matrix with the singular values;
$\mathbf{V^T}$ is the transpose of a unitary column-orthogonal matrix, $\mathbf{V}$.

The singular values obtained along the diagonal of $\mathbf{\Sigma}$ are also the principal components (PC). The PC are defined such that the one with the largest variance within the data is the first PC (the first singular values), the next most varying is the second PC and so on. The smallest singular value thus corresponds to the weakest PC that has little to no variation. A matrix of PC scores can then be constructed as:

$$\mathbf{Z} = \mathbf{XV} = \mathbf{U\Sigma}. \tag{8}$$

The contribution of each PC can be evaluated by dividing $\mathbf{Z}$ with the sum of all the PC scores. For each PC, this yields a value between 0 to 1. The larger the number, the more significant the PC is. In other words, a weakly contributing PC can be identified and thus be removed by setting it to zero as they are mainly caused by noise influences or external disturbances [28]. Then, the inverse of the singular value matrix $\mathbf{\Sigma}^{-1}$ can be recalculated by:

$$\mathbf{\Sigma}_e^{-1} = \begin{cases} 1/\sigma_q & \text{if } \overline{\sigma}_q \geqslant \theta, \\ 0 & \text{otherwise,} \end{cases} \tag{9}$$

where σ_q indicates the q-th singular value along the diagonal $\mathbf{\Sigma}$ matrix and the overbar indicates normalised singular value against sum of all singular values [26]. On the other hand, θ represents a threshold value (where $0 \leqslant \theta \leqslant 1$) while the subscript e indicates that the matrix has been enhanced by SVD and PCA.

Subsequently, the modified pseudo-inverse of $\mathbf{X}$ can be written as [34]:

$$\mathbf{X_e^{+}} = \mathbf{V\Sigma}_e^{-1}\mathbf{U^T}. \tag{10}$$

Introducing Eq. (10) into Eq. (6), the treated transmissibility matrix $\mathbf{H_e}$ can then be written as:

$$\mathbf{H_e} = \mathbf{X_e^{+}Y}. \tag{11}$$