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Linear approximation of underwater sound speed profile: Precision analysis in direct and inverse problems

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ABSTRACT

Sound speed is one of the key sources of uncertainty in an underwater environment. The wave equation for a complex sound speed profile (SSP) cannot be analytically solved and numerical solutions generally involve high computational costs. A relatively simple way is to approximate the SSP by assuming it as horizontally stratified, vertically multi-layered and linearly varied with respect to water depth in each layer. This approximative model leads to a fast computation of the sound propagation. However, the error of SSP results in an inaccurate sound field computation, thus the model accuracy in terms of error propagation in both direct and inverse problems should be investigated. Rapidly developing numerical techniques are currently able to accurately simulate the sound propagation in a complex configuration, such that the difference between a real case with a complex SSP and its approximation can be precisely quantified. In this paper, the sound propagation with a complex SSP is simulated via a full wave numerical approach, known as the spectral element method. The efficiency of SSP linear approximation with various layer number (corresponding to different sound speed error) is quantified via transmission loss forecast (direct problem) and sound source localization error (inverse problem), respectively. The precision analysis is able to guide the choice of optimal approximate model for different scenarios, which is a trade-off between the computational cost and the model accuracy.

1. Introduction

The sound field forecast and source localization in underwater environments are challenging problems due to the complexity of the sound propagation medium [1–3]. The complex nature of sound speed is one of the main reasons, which may be inhomogeneous and imprecisely measured [4–9]. Although in many cases the water column and the seabed can be reasonably assumed as horizontally stratified, the sound speed in the vertical direction typically varies in a broad range, affected by temperature, salinity, etc. In order to simulate the sound propagation in a realistic medium with a complex sound speed profile (SSP), numerical methods, such as finite-difference, boundary-element, classical low-order finite-element methods or spectral-element method, can be used [2,10]. However, simulating a high frequency wave in a long range involves a high computational cost. In particular, source localization problems usually require a vast number of sound propagation computations. Therefore, a rapid sound field computation method is desirable.

In order to simplify the sound propagation model, a linearly varied SSP can be considered. Planar multi-layered medium is considered in Refs. [11–14] where a general solution of the depth-dependent wave equation can be given. The Helmholtz equation can be computed using the wavenumber integration [11,12] and normal mode [15,16,13] methods. However, the approximation of SSP results in an error of sound field computation which needs to be evaluated.

In this article, the underwater region is assumed to be planar multi-layered, i.e., horizontally stratified and vertically divided into multiple layers. In addition, in each layer the sound speed is constant or pseudo-linear (the square of the wavenumber linearly varies) with respect to the water depth. The sound field is computed via the wavenumber integration method [11,2]. The efficiency of this sound speed approximation to a realistic case (with a complex SSP) in the sense of sound field forecast and source localization accuracy is quantified. For this purpose, the results obtained from the multi-layer linear sound speed model is compared with a realistic SSP model; the latter is realized via a time domain full wave numerical method, known as the spectral-element method (SEM) [17,18]. This approach is a formulation of the finite-element method that uses high-degree polynomials as elemental basis functions. The accuracy of this numerical method for sound wave simulation in a complex underwater environment, with complicated

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SSP, different sediment, irregular layer interface and reflections, etc., has been validated [10]. In the present paper, the considered underwater region is in 3D but the 2D version of SEM can be used since the wave equation is assumed to be independent of horizontal azimuth. More specifically, the time-domain sound wave is simulated via the open-source code SPECFEM2D based on an axisymmetric formulation [19].

In ocean acoustics, the computation error in both direct and inverse problems are concerned. The former means the forecast of sound field: it can be quantified by the transmission loss, which describes the accumulated decrease of sound energy as a wave propagates outwards from a source. The latter implies the accuracy of sound source localization. In this work, the matched-field processing (MFP) method [20–23] is used for the source localization problem. This spectral-based signal processing technique matches the field distribution versus the source location parameter. Conventional MFP [24,25] maximizes the power output of a point source and leads to a maximum likelihood estimate (MLE) of the source location and strength [26–28]. Minimum variance distortionless filter (MVDF), or Capon's, MFP [29,30] minimizes the variance at the output of a linear weighting of the sensors subject to unit gain. Compared to the conventional MFP, the MVDF approach results in a super-resolution estimation and compresses local maxima of objective function, but is more sensitive to model uncertainties [20,31]. Furthermore, it requires a large number of data since the sample covariance matrix has to be computed while the conventional MFP works even if only one snapshot is available. In this paper, the errors of transmission loss forecast and MFP source localization are both computed, using the linearized multi-layer model with various number of layers. Actually, as the number of layers increases, both errors decrease since the sound speed is better fitted, but the computational cost increases as well. The model precision analysis in this paper provides a guidance for choosing an optimal model in terms of layer number, which is a trade-off between the computational cost and error.

The organization of the paper is as follows. Section 2 introduces the sound propagation computation with a multi-layer linear SSP, which is used to approximate a realistic underwater medium. In Section 3, the approximation efficiency of the linearized SSP is quantified in both direct and inverse problems via sound field forecast and source localization respectively. Then, a numerical example of summer Mediterranean with a complex SSP is considered in Section 4. The SEM, which is used for simulating wave propagation in the complex medium, is introduced. The errors in both the direct and inverse problems due to the linear approximation of SSP are analyzed. Finally, conclusions are drawn in Section 5.

2. Sound propagation in underwater with a multi-layer linear sound speed profile

In this section, the sound propagation model assuming multi-layer linear SSP is considered. The underwater region $\{\mathbf{r}=(x,y,z): z\in[d_0,d_L]\}$, where $d_0=0$ and $d_L=d$ respectively stand for the water surface and the water bottom, is horizontally stratified and divided into L layers with interfaces $d_1,\cdots,d_{L-1}$, as shown in Fig. 1. The sound source $\mathbf{r}_0=(x_0,y_0,z_0)$ is assumed to be in the s-th layer, i.e. $z_0\in(d_{s-1},d_s],s\in\{1,\cdots,L\}$. The sound speed in the water column is continuous with respect to the depth z and is pseudo-linear (the square of the wavenumber is varied linearly with respect to the depth) in each layer. The sound speed at each interface is

$$v(d_l)=v_l,l=0,1,\cdots,L \tag{1}$$

and in the l-layer is thus

$$v_l(z)=\sqrt{\frac{1}{a_lz+b_l}},z\in[d_{l-1},d_l], \tag{2}$$

in which

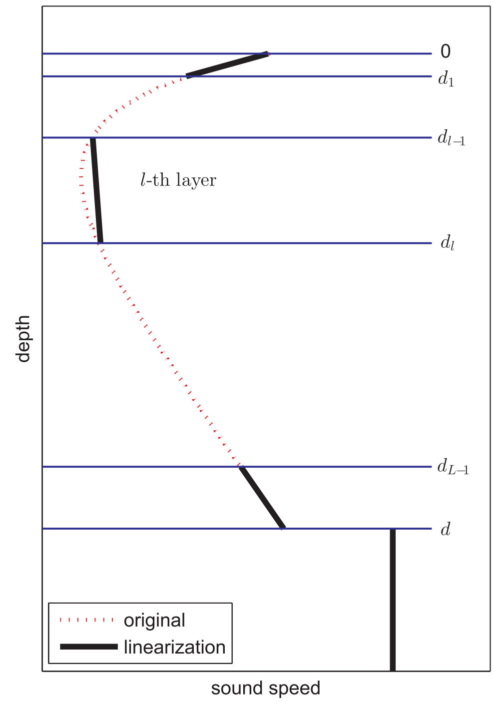

Fig. 1. Multi-layered linear approximation of sound speed profile.

$$a_l=\frac{v_{l-1}^2-v_l^2}{v_l^2v_{l-1}^2(d_l-d_{l-1})},\ b=\frac{v_{l-1}^2d_{l-1}-v_l^2d_l}{v_l^2v_{l-1}^2(d_l-d_{l-1})}. \tag{3}$$

The seabed is represented by an infinite fluid halfspace $\{\mathbf{r}=(x,y,z): z\in[d,\infty)\}$, in which the sound speed is constant and represented by v_{L+1}. The density in the water column and the ocean bottom is assumed to be constant:

$$\rho(z)=\begin{cases}\rho_1, & z\in[0,d]\\ \rho_2, & z\in(d,\infty)\end{cases}. \tag{4}$$

The wave equation for the displacement potential as a function of the spatial coordinate $\mathbf{r}=(x,y,z)$ and time t is governed by

$$\left(\nabla^2-\frac{1}{v^2(z)}\frac{\partial^2}{\partial t^2}\right)\widetilde{\psi}(\mathbf{r},t)=\delta(\mathbf{r}-\mathbf{r}_0)S_t, \tag{5}$$

in which S_t is a deterministic function of source signal in the time domain, and δ is the Dirac delta function. Taking a Fourier transform of both sides of Eq. (5) with respect to t results in the Helmholtz equation for the sound field $\psi(\mathbf{r},f)$ in the frequency domain:

$$(\nabla^2+k^2(z))\psi(\mathbf{r},f)=\delta(\mathbf{r}-\mathbf{r}_0)S_f, \tag{6}$$

where f is the frequency and S_f is the Fourier transform of S_t. The sound pressure is obtained from the displacement potential as

$$p(\mathbf{r},f)=\rho\omega^2\psi(\mathbf{r},f), \tag{7}$$

where $\omega=2\pi f$ is the angular frequency.

Assuming that the sound source is omni-directional, thus the sound field only depends on the depth and the horizontal range. Then, a cylindrical coordinate system is chosen, i.e., the spatial coordinate is denoted by $\mathbf{r}=(r,z,\phi)$, in which r and ϕ stand for the horizontal range and azimuth, respectively. By applying the Hankel transform

$$\psi(k_r,z)=\int_0^\infty\psi(r,z)J_0(k_rr)rdr \tag{8}$$

to Eq. (6), the depth-separated wave equation is obtained:

$$\left[\frac{\partial^2}{\partial z^2}+(k^2(z)-k_r^2)\right]\psi(k_r,z)=S_f\frac{\delta(z-z_0)}{2\pi}. \tag{9}$$