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# Analysis of mode mismatch in uncertain shallow ocean environment

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ABSTRACT

The actual ocean is an uncertain acoustic propagation environment. For the localization algorithms that rely on the precise ocean environmental parameters, the environmental mismatch problems will exist and performance degradation may be very serious. In the uncertain ocean environment, the uncertainties of sound field will have different effects on different normal modes propagating in the sound field, thus analysis of the mismatch characteristics of normal modes affected by uncertain environmental parameters can provide technical guidance for practical engineering applications. Based on the shallow-water acoustic propagation model, this paper simulates and analyzes the mismatch results of the modal depth eigenfunction and the horizontal wave number of each normal mode under conditions of environmental mismatch. Research indicates that the influence of different environmental parameters on normal modes in the sound field is not exactly the same. It was found that the sound-speed profile and seawater depth affect significantly, followed by sediment sound speed, the other parameters appear to be relatively minor importance.

## 1. Introduction

The real ocean environment is a complex acoustic propagation channel which is time-varying and space-varying, due to the influence of wind waves, internal waves and suspended solids in seawater. The conventional matched-field methods will have the environmental mismatch problems when the target is locating, resulting in the degradation of the localization performance. There have been many researches on this issue.

The effect of incorrect estimates of the water column depth on matched field source localization in a shallow-water environment was discussed [1] and it was found that significant errors can be introduced into the range and depth localization predictions of a matched-field processor through erroneous estimates of the water depth. The effects of variations in geoacoustic environmental parameters on the performance of a matched-field localization processor in shallow water were investigated [2,3]. It was found that small perturbations in a downward-refracting summer water sound-speed profile caused severe degradation in localization performance, with predictions of source range and depth becoming highly unstable. A. Tolstoy [4] examined the sensitivity of matched-field processing to the sound-speed profile mismatch based upon archival profile resulting in various degrees of mismatch. And it came to the conclusion that the matched-field processing can be very sensitive to the sound-speed profile mismatch, but that the degree of sensitivity to a given mismatch is strongly affected by array parameters, i.e., number of phones and array depth. The source localization performance degradation of the matched-field processing caused by environmental mismatch has been discussed in mode space analytically and analytical results have been verified by numerical simulations [5]. The performance sensitivity of broad-band MV_MFBF to eight ocean environmental parameters (water sound speed, depth etc.) was discussed based on the typical shallow water environmental benchmark set by the NRLWorkshop_93 [6]. The performance was measured quantitatively by three parameters: the location bias, the ambiguity surface peak of an uncertain case to a peak of a certain case ratio and the peak to the background ratio of the ambiguity surface.

For the extremely sensitive phenomenon to errors of localization process in the assumed environmental conditions, several approaches to this problem aiming at stable target localization have been investigated. Representative methods include the reduced minimum variance beamformer (RMV), the minimum variance beamformer with neighborhood location constraints (MV_NLC), the minimum variance beamformer with environmental perturbation constraints (MV_EPC), MinMax [7–9], the maximum likelihood (ML) [10]. These methods can

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be categorized into the adjacency constraint algorithms. The basic principle is to constrain the response of the surrounding adjacency position in the case of ensuring that the desired response is constant to achieve the purpose of robust processing, which is also widely used in array signal processing. The optimum uncertain field processor (OUFP) [11] uses a computationally intensive integration over the model parameter space to eliminate the effects of uncertain parameters. This method has the best localization performance in the statistical sense, but its obvious shortage is that the integral operation is very complicated.

In the uncertain marine environment, the modal function of the normal mode appears random fluctuation. However, some modal amplitude remain more correlated than others in the presence of environmental uncertainties, for example, the uncertainty of the boundary interactions is one of the main perturbation sources of the underwater acoustic propagation. In general, the higher-order modes correspond to more sound rays, boundary coupling and are thus less predictable. In typical shallow-water channels, the higher-order modes are associated with rays that have a higher number of reflections from the boundaries and larger group delays in the medium. Therefore, these modes are often affected by medium variations more than the lower-order modes. The lower-order modes correspond to less sound rays and boundary coupling, which means that the lower-order mode carries more information about the target location. Based on this, Tabrikian, et al. decomposed the sound field into predictable and unpredictable subspaces of the acoustic normal mode representation. The estimator uses the predictable subspace for source localization [12]. Liu et al. further proposed a robust localization method by mode subspace reconstruction [13].

The researches above mainly pay attention to the influence of localization performance by the uncertain environment which is the analysis to the localization results. This paper focuses on the effect of uncertain environmental parameters on the mode itself. We use the typical shallow water environmental benchmark provided by NRLWorkshop_93 as the test model and the matched-mode processing is used to obtain each independent mode. The mismatch characteristics of the modal depth eigenfunction and the horizontal wave number of each normal mode under conditions of environmental mismatch are simulated and analyzed respectively.

## 2. Theoretical model

### 2.1. Adiabatic normal mode model

Consider a point source at depth $z_s$ and range $r_s$, which radiates a monochromatic signal at angular frequency $\omega$ in a time invariant shallow-water waveguide. The density of water is $\rho$. The sound field $p(\mathbf{r})$ at $\mathbf{r} = (r,z)$ satisfies the Helmholtz equation

$$\nabla p(r,z) + k^2(r,z)p(r,z) = -4\pi\delta(r)(z-z_s) \tag{1}$$

where $k(r,z) = \omega/c(r,z)$. For the far-field low-frequency case, the adiabatic normal mode model can quickly and effectively computes the radiation field, and the field measured by sensor in Eq. (1) is

$$p(r,z) = \frac{je^{j\pi/4}}{\rho(z_s)\sqrt{8\pi r}}\sum_{m=1}^{M}\phi_m(z_s)\phi_m(z)\frac{e^{k_{rm}r}}{\sqrt{k_{rm}r}} \tag{2}$$

Here, $M$ is the number of the propagating modes in the channel. $\phi_m(z_s)$ and $\phi_m(z),m = 1,\ldots,M$ are the modal depth eigenfunctions at the source and the array, respectively, and $k_{rm}$ denotes the horizontal wave number of the $m$-th mode in the channel. The modal eigenfunctions, $\phi_m(z_i)$, and horizontal wave numbers $k_{rm}$ , depend on environmental parameters which describe the bathymetry, geoacoustic properties of the bottom, and sound speed in the water column. In practice, these parameters are not precisely known. Eq. (2) can be written as

$$p(r,z) = \sum_{m=1}^{M} a_m(r,z_s)\phi_m(z) \tag{3}$$

where the modal coefficient $a_m(r,z_s)$

$$a_m(r,z_s) = \frac{je^{j\pi/4}}{\rho(z_s)\sqrt{8\pi r}}\frac{e^{k_{rm}r}}{\sqrt{k_{rm}r}}\phi_m(z_s) \tag{4}$$

The sound field is sampled by a vertical array of $N$ sensors and the depth of the $i$-th sensor from the upper surface is denoted by $z_i$. Then the received sound pressure signal of vertical array is

$$p = \begin{bmatrix} p(r,z_1) \\ \vdots \\ p(r,z_N) \end{bmatrix} = \begin{bmatrix} \phi_1(z_1)\cdots\phi_M(z_1) \\ \vdots\ \ddots\ \vdots \\ \phi_1(z_N)\cdots\phi_M(z_N) \end{bmatrix}\begin{bmatrix} a_1 \\ \vdots \\ a_M \end{bmatrix} = \Phi\mathbf{a} \tag{5}$$

where $z_i,i = 1,\ldots,N$ is the depth of $i$-th sensor, $\Phi$ and $\mathbf{a}$ represent the modal matrix and modal coefficient vector. For the single radiated source, set $a_0$ as complex amplitude and the noise is $n_0$, the received data can be written as

$$r = a_0\Phi\mathbf{a} + n_0 \tag{6}$$

### 2.2. Matched-mode processing

The basic principle of matching-mode processing (MMP) is to convert the array domain data into the modal domain for processing [14,15] and the advantage is that it is possible to selectively process each mode separately for better processing gain based on some characteristics of the sound field itself.

The MMP first decomposes the received data and converts the signal from the array domain to the modal domain, then matches the modal coefficient and the replica field. The modal decomposition is to estimate the modal coefficient vector $\mathbf{a}$ in the case of knowing array receiving data $r$ and the modal matrix $\Phi$. It is a linear inverse problem as in Eq. (7).

$$\widetilde{a} = Gr \tag{7}$$

where $\widetilde{a}$ is the estimated modal coefficient and $G$ is the $M \times N$ modal inverse filter. The filter has a variety of construction methods, including modal sampling modal filters [16], pseudo inverse modal filters [17] and overall least squares modal filters [18]. In this paper, pseudo-inverse modal filter is used. Under the condition of space under-sampling, Eq. (7) is often an underdetermined equation set and $G$ is not a square matrix, therefore, we need to compute the generalized inverse matrix of $\Phi$ to obtain the minimal norm quadratic solution of $\mathbf{a}$. That is

$$G^{+} = (\Phi^H\Phi)^{-1}\Phi^H \tag{8}$$

The modal covariance matrix is

$$R_M = GRG^H \simeq \langle\widetilde{a}\widetilde{a}^H\rangle \tag{9}$$

The output ambiguity surface of MMP processor is

$$B(\widetilde{r},\widetilde{z}) = \hat{a}^H(\widetilde{r},\widetilde{z})R_M\hat{a}(\widetilde{r},\widetilde{z}) \tag{10}$$

where $\hat{a}(\widetilde{r},\widetilde{z})$ is the modal coefficient vector of normalized replica field. $B(\widetilde{r},\widetilde{z})$ is the MMP output power at scanning position $(\widetilde{r},\widetilde{z})$. In the case of only one target, the corresponding position of the maximum value of $B$ can be used as an estimate of the target position.

## 3. Simulation experiment and result

The environmental scenario we used in this paper is one of the more complex benchmark cases used in the May 1993 NRL Workshop on Acoustic Models in Signal Processing [19]. Fig. 1 depicts the environmental configuration and the source-array geometry. Where, the water depth is 102.5 m, the sound-speed profile is linear negative gradient. The sound-speed profile of sediment layer within 200 m is linear positive gradient and is constant outside 200 m which is lower halfspace