# The caustic in the acoustics of historic interiors

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## ARTICLE INFO

## ABSTRACT

In the paper, caustics are discussed as ordered forms of focus broadening occurring in concave mirrors. Presented are examples of caustics formed in natural conditions and application of the phenomenon in different fields of science and technology. Against this factual background, the possibility to observe caustics in rooms is pointed out. Special emphasis is put on large rooms of historic character, as such interiors frequently include acoustic mirrors in the form of arched vaults and concave walls. As a case study, the eighteenth-century Whispering Grottoes representing one of attractions of the Oliwa Park in Gdańsk was selected, where the phenomenon of forming a caustic was used intentionally to obtain the desired acoustic effect.

## 1. Introduction

In large historical interiors formed by curved surfaces, it is very likely to come across acoustical singularities consisting in "sliding" of sound along a concave wall or concentration of sound in a distant location of the room. The physical nature of these singularities may be different but the effect is the same—two people can hear each other over a large distance without using any additional sound amplification systems.

In contemporary auditoriums, these phenomena are considered an acoustic flaw and are usually effectively corrected. However, the present study deals with rooms created at a time when room acoustics were not yet a scientific discipline in today's sense. The discussed singularities did not subject to acoustic correction, and even their existence was not always noticed.

They can be mostly found in historical premises serving stately, ceremonial, liturgical, or similar purposes where these effects usually were detected accidentally, sometimes many years after erection of the building. Descriptions of these places of interest can be found in books published as early as in the beginnings of development of the architectural acoustics as a scientific discipline [1–4].

The aim of this paper is to prove that the discussed phenomena occur as a result of formation of a caustic, being a specific form of blurring the focus of the rays. The concept of caustics with respect to optical mirrors is known for several centuries ("One of the earliest discoveries in optics (F. Maurolycus, 1575) was that the rays of a normal system are tangential to a surface, the so-called caustic surface." [5]). However, despite the close relationships between room acoustics and the principles of geometric optics, the term "caustics" in the developed form is in principle absent in the contemporary literature on room acoustics. The paper discusses in detail the 18th-century historic objects, which have been built from scratch to demonstrate the whispering wall phenomenon to the public, with intentional using the caustic for this purpose.

## 2. Geometrical model of a sound field in rooms

In this paper, a general assumption of geometrical room acoustics is made that all structures are large compared to the wavelength (so called "optical limiting case"). In this model, the field is composed of sound rays which do not subject to the phenomenon of diffraction. However, in a physical field, the phenomenon of diffraction can significantly influence the form of the field. One of the effects of sound diffraction in rooms is the "recognition" of the obstacle details by the wave front [6]. This reaction is the more accurate, the smaller is the wavelength. This means that waves long enough do not "see" the details of the obstacle and are reflected as from a flat surface.

For the specular reflection, the relationship between the wavelength and the dimensions of the obstacle itself and its elements, i.e. details of its surface, depth of the curvature of the obstacle, etc. is given in Eq. (1)

$$l \leqslant K\lambda \tag{1}$$

where $\lambda$ is the length of the longest reflected wave reflected in a specular way; $l$ is the smallest dimension of the obstacle or the size of its detail; and $K$ is a factor dependent on the ratio between mirrored or diffused energy.

In the literature, the value of $K$ is generally larger than 11. This means that for a geometrical reflection, the obstacle with its details should be greater than the wavelength, according to some authors up to 2 times [7] or even more (e.g. $K > 4$ for the screen itself, $K > 1$ for the
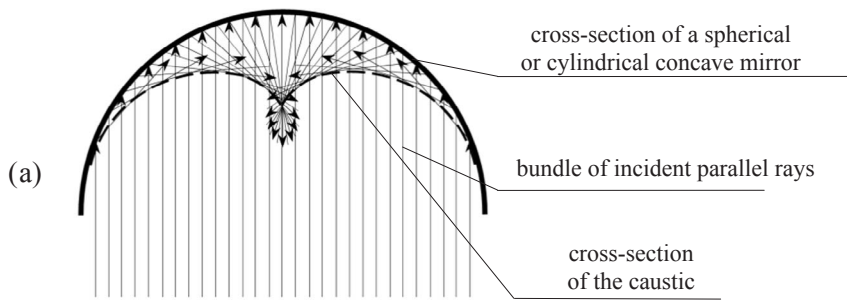
**Fig. 1.** Catacaustic as a manifestation of spherical aberration of spherical or cylindrical concave mirror: (a) geometrical representation of catacaustic; (b) physical counterpart of the catacaustic drawn in panel (a) [18]; (c) computer simulated superposition of a catacaustic and a diacaustic as a result of light refraction at the boundary between two media and reflection on the inner surface of a vessel [19].
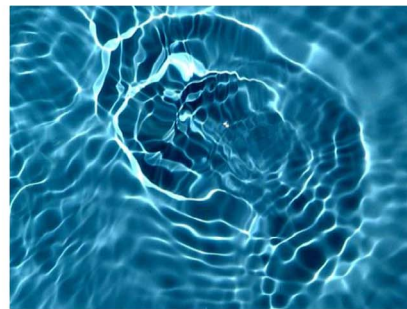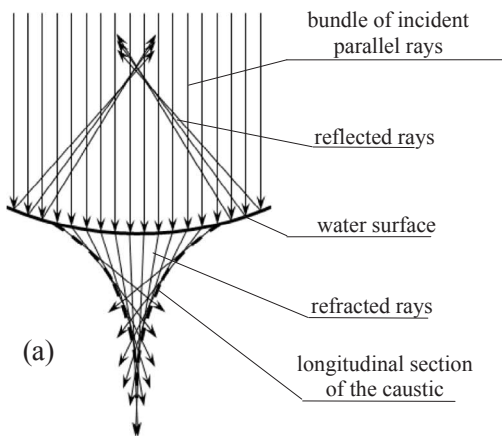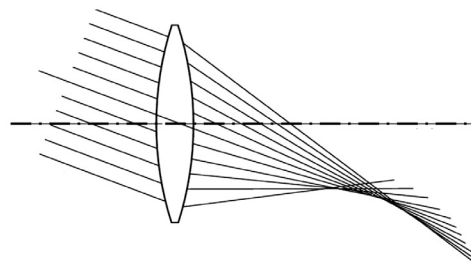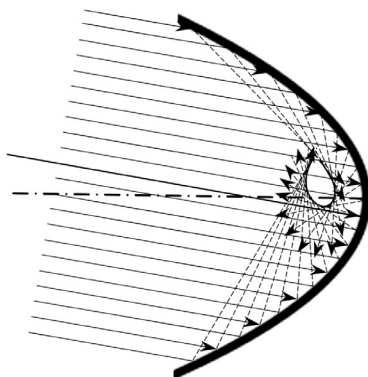


**Fig. 2.** The diacaustic as an effect of refraction of light on rippled water surface: (a) geometrical representation of a diacaustic in longitudinal section; (b) physical embodiment of a diacaustic cross-section [18].



**Fig. 3.** Longitudinal section of (a) catacaustic and (b) diacaustic [11] being manifestations of coma (comatic aberration) in a parabolic concave mirror and a converging lens. The focus stretches out when the source of rays is situated outside the axis of symmetry.

screen details [8]). Also, $K$ values less than 1 can be found, where reflection is still considered to be specular with much larger component of the scattered energy ($K = 1/3$, [9]).

Apart from the diffraction, the phenomenon of wave interference is also observed in the real acoustic field. Therefore, real focuses and caustics have a form of an area of a size depending on the wavelength [10]. This work is based on geometrical model and all considerations are valid only for the "optical limiting case", where focuses are represented by points and cross-sections of the caustics are represented by lines.