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A finite-element method using dispersion reduced spline elements for room acoustics simulation

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A B S T R A C T

This paper presents a finite element method (FEM) using hexahedral 27-node spline acoustic elements (Spl27) with low numerical dispersion for room acoustics simulation in both the frequency and time domains, especially at higher frequencies. Dispersion error analysis in one dimension is performed to increase the accuracy of FEM using Spl27 by modifying the numerical integration points of element stiffness and mass matrices. The basic accuracy and efficiency of the FEM using the improved Spl27, which uses modified integration points, are presented through numerical experiments using benchmark problems in both the frequency and time domains, revealing that FEM using the improved Spl27 in both domains provides more accurate results than the conventional method does, and with fewer degrees of freedom. Moreover, the effectiveness of FEM using the improved Spl27 over that using hexahedral 27-node Lagrange elements is shown for time domain analysis of the sound field in a practical sized room.

1. Introduction

The finite element method (FEM) is a physically reliable numerical method based on wave acoustics for room acoustics simulation in both frequency and time domains [1–4]. Because FEM is frequently said to be computationally expensive for room acoustics simulation with complex boundary conditions, application of the method is restricted to low-frequency regions in general, but the situation is changing quickly with the rapid progress of computer technology and with the development of efficient methods. Therefore, the use of the method has recently become a realistic option to predict the sound field in an architectural space at the high-frequency region up to some kilohertz.

The authors have developed an efficient FEM [5–9] using high-order elements, namely, hexahedral 27-node spline acoustic elements (Spl27) [10,11], preconditioned iterative methods, and parallel computation techniques, to predict large-scale sound fields in rooms with many degrees of freedom (DOF) accurately and efficiently in both frequency and time domains. Here, we define frequency domain FEM and time domain FEM respectively as FD-FEM and TD-FEM. Using the methods, several sound fields in rooms such as concert halls and reverberation chambers have been predicted at low frequencies below one kilohertz, where accuracies were examined by comparison with measurements or other numerical methods [6,12–14].

An issue of great concern in finite element (FE) analysis of acoustics is associated with the efficient prediction of sound fields at high frequencies in some kilohertz ranges with reliable accuracy. That issue is reduction of the discretization error, called dispersion error, which is defined as the difference between the exact wave number and numerical wave number or between exact wave velocity and numerical wave velocity. Because of the error, a spatial discretization requirement is imposed in the mesh generation process. For time domain analysis, time discretization error must also be considered in order to yield reliable results. Because these requirements engender a marked increase of computational cost in analysis at high frequencies, many methods have been proposed to reduce the dispersion error [10,15–17]. A useful review [15] presents methods for reducing the error in spatial discretization.

Among the methods, there exist a simple but surprisingly efficient method for low-order elements, called modified integration rules (MIR) [16,18], for reducing the dispersion errors in both frequency and time domain analyses. By simply changing numerical integration points of element matrices from conventional points in standard FEM using four-node quadrilateral elements, the resulting FEM has fourth order accuracy with respect to dispersion error, whereas standard FEM has second-order accuracy. Therefore, the use of MIR can reduce the computational cost markedly to yield similarly accurate results as the standard FEM with low

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dispersion error. Recently, we have applied MIR to TD-FEM using hexahedral eight-node elements and an iterative method for room acoustics simulation [19]. The accuracy and efficiency of the TD-FEM using MIR in three dimensions was presented over conventional TD-FEM through three-dimensional dispersion error analysis and numerical experiments, in which we also revealed that the use of MIR improves the convergence of an iterative method to a marked degree.

When modeling sound fields with curved surfaces, discretization of the computational domain using high-order FEs is more efficient than that using first-order FEs such as eight-node hexahedral elements from the perspective of accuracy in geometrical modeling. Furthermore, because the use of the high-order FEs generally produces much more accurate results than first-order FEs does, and with fewer DOF, development of more accurate and efficient FEM using high-order FEs is beneficial to predict sound fields in rooms with complex boundary conditions at high frequencies.

Therefore, the idea of MIR is applied herein to FEM using Spl27 as high-order elements. As a consequence, we propose FEM having improved Spl27 that uses modified integration points in numerical integrations of element matrices based on dispersion relation in one dimension. First, we briefly describe theories of FD-FEM and TD-FEM for sound field analysis and Spl27. Secondly, a method to increase the accuracy of FEM using Spl27, which is based on one-dimensional dispersion relation is presented, particularly addressing reduction of only spatial discretization error. Furthermore, the accuracy in one-dimensional analysis over FEM using conventional spline elements is theoretically estimated as showing the dispersion errors for both methods as a function of spatial resolution. We present the basic accuracy and efficiency of FD-FEM and TD-FEM using the improved Spl27 over conventional method assisted by numerical experiments using benchmark problems in the frequency up to 4 kHz. Finally, the effectiveness of TD-FEM using Spl27 over standard Lagrange elements is demonstrated further.

2. Theory

2.1. FEM for sound field analysis in frequency and time domains

The FE equation in the frequency domain for a three-dimensional sound field (air density, ρ; speed of sound, c) with impedance boundaries and with vibration boundaries, is derived from the principle of minimum potential energy as

$$(\boldsymbol{K} - k^2\boldsymbol{M} + ik\boldsymbol{C})\boldsymbol{p} = i\omega\rho v_{\mathrm{n}}\boldsymbol{W}, \tag{1}$$

where $\boldsymbol{K}, \boldsymbol{M}$, and $\boldsymbol{C}$ respectively represent the global stiffness matrix, global mass matrix, and global dissipation matrix. $\boldsymbol{p}$ is the sound pressure vector and $\boldsymbol{W}$ is the distribution vector. $k, \omega, v_{\mathrm{n}}$ and i respectively denote the wave number, the angular frequency, the velocity of vibration and the imaginary unit. The respective global matrices $\boldsymbol{K}, \boldsymbol{M}$, and $\boldsymbol{C}$ are constructed from the respective element matrices defined as follows:

$$\boldsymbol{K_e} = \int_{\Omega_e} \nabla\boldsymbol{N}^{\mathrm{T}}\nabla\boldsymbol{N}d\Omega, \tag{2}$$

$$\boldsymbol{M_e} = \int_{\Omega_e} \boldsymbol{N}^{\mathrm{T}}\boldsymbol{N}d\Omega, \tag{3}$$

$$\boldsymbol{C_e} = \frac{1}{z_{\mathrm{n}}}\int_{\Gamma_e} \boldsymbol{N}^{\mathrm{T}}\boldsymbol{N}d\Gamma, \tag{4}$$

with the normalized acoustic impedance ratio z_{n} and shape function $\boldsymbol{N}$. Ω_e and Γ_e respectively represent the region and surface areas of an element to be integrated. $\boldsymbol{p}$ at an ω is obtainable by solving the linear system of equations of Eq. (1) using a direct method or an iterative method.

FE formulation in the time domain of Eq. (1) can be written as

$$\boldsymbol{M}\ddot{\boldsymbol{p}} + c^2\boldsymbol{K}\boldsymbol{p} + c\boldsymbol{C}\dot{\boldsymbol{p}} = \rho c^2\dot{v}_{\mathrm{n}}\boldsymbol{W}. \tag{5}$$

$\boldsymbol{p}$ in time domain is calculable using a direct time integration method such as Newmark β method [20]. Herein, a method in the Newmark family called Fox–Goodwin method [21] is used for the time integration. The stability condition of the Fox–Goodwin method is given as follows.

$$\Delta t_{\mathrm{crit.}} \leqslant \frac{1}{\omega_{\max}\sqrt{1/6}}. \tag{6}$$

Therein, $\Delta t_{\mathrm{crit.}}$ is the critical time interval. The maximum natural frequency of system $\omega_{\max}$ is obtainable by solving a generalized eigenvalue problem $(\boldsymbol{K}_e - \omega^2\boldsymbol{M}_e)\boldsymbol{p}_e = 0$. Here, $\boldsymbol{p}_e$ is sound pressure vector within an element.

2.2. Spline acoustic elements

The Spl27 [10,11] is hexahedral 27-node isoparametric FEs using the natural cubic spline polynomial function S_i for $\boldsymbol{N}$. The shape function for Spl27 in three dimensions is defined as

$$\boldsymbol{N}_m(\xi,\eta,\zeta) = S_i(\xi)S_i(\eta)S_i(\zeta)(m = 1,2,\cdots,27), \tag{7}$$

with

$$(\text{if } \xi_i = \pm 1)S_i(\xi) = \begin{cases} 0.25\xi^3 + 0.75\xi^2 + 0.5\xi_i\xi & : \xi \in [-1,0] \\ -0.25\xi^3 + 0.75\xi^2 + 0.5\xi_i\xi & : \xi \in [\;0,1] \end{cases} \tag{8}$$

$$(\text{if } \xi_i = \;0)S_i(\xi) = \begin{cases} -0.5\xi^3 - 1.5\xi^2 + 1 & : \xi \in [-1,0] \\ 0.5\xi^3 - 1.5\xi^2 + 1 & : \xi \in [\;0,1] \end{cases} \tag{9}$$

Here, ξ,η,ζ represent the coordinates of a hexahedron in a local coordinate system. ξ_i is the local corner coordinate of the hexahedron in the ξ-direction. For the η-direction and ζ-direction, the function forms of S_i are identical.

3. Dispersion reduced spline acoustic elements

3.1. Dispersion error analysis in one dimension

The method described in **3.2.**, which was recently proposed by the authors in a letter [22], uses a dispersion relation in one dimension to increase the accuracy of FEM using Spl27. Here, the dispersion relation between exact wave number k and numerical wave number k^h is derived using a dispersion error analysis in one dimension.

The dispersion error $e_{\mathrm{dis.}}$ is defined as

$$e_{\mathrm{dis.}} = \frac{|k^h - k|}{k}. \tag{10}$$

In the equation presented above, k^h for evaluating $e_{\mathrm{dis.}}$ can be derived analytically, using a FE mesh discretized by three-node spline line elements of nodal distance d, as presented in Fig. 1.

The element matrices $\boldsymbol{K}_e$ and $\boldsymbol{M}_e$ for the three-node spline line elements are calculable using the Gauss–Legendre rules with three numerical integration points as

$$\boldsymbol{K}_e = \sum_{i=1}^{3} W_i\nabla\boldsymbol{N}(\xi_i^K)^{\mathrm{T}}\nabla\boldsymbol{N}(\xi_i^K)\det(\boldsymbol{J}), \tag{11}$$

$$\boldsymbol{M}_e = \sum_{i=1}^{3} W_i\boldsymbol{N}(\xi_i^M)^{\mathrm{T}}\boldsymbol{N}(\xi_i^M)\det(\boldsymbol{J}), \tag{12}$$

with a shape function defined as