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Thermo-physical aspects in tangent hyperbolic fluid flow regime: A short communication

Khalil Ur Rehman[b,*], Ali Saleh Alshomrani[a], M.Y. Malik[a,b], Iffat Zehra[c], Muhammad Naseer[b]

[a] Department of Mathematics, Faculty of Science, King Abdulaziz University, PO Box-80203, Jeddah 21589, Saudi Arabia
[b] Department of Mathematics, Quaid-i-Azam University, Islamabad 44000, Pakistan
[c] Department of Mathematics, Air University, PAF Complex E-9, Islamabad 44000, Pakistan

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ABSTRACT

The present attempt is made to report the flow regime characteristics of tangent hyperbolic fluid when both the magnetic field and heat generation effects are taken into account. The flow narrating differential equations subject to thermally stratified medium are transformed into a system of nonlinear ordinary differential equations. A computational algorithm is developed to offer a numerical solution of the flow problem. The physical outcomes against flow controlling parameters namely, curvature parameter, Weissenberg number, power law index, thermal stratification, heat generation and Prandtl number are discussed and illustrated via graphs and tables. The outcomes are certified by providing comparison with existing literature in a limiting sense.

1. Introduction

The examination of magnetohydrodynamic (MHD) boundary layer flow over stretching surfaces has significant importance in industrial and manufacturing processes namely electrical power generators, magnetic cell separation (MCS), pumps, magnetic resonance imaging (MRI) and drugs transportation through magnetic particles as a drug carrier to mention just a few. Such type of applications motivate researchers to trace out the exact and numerical solutions of MHD fluid flow narrating differential equations like Pavlov [1] discussed the magnetohydrodynamic fluid flow past a stretching surface and offered exact solution of momentum equation as a pioneer contribution. Later on, numerous attempts are reported on stretching flat surface along with different physical effects, one can assessed in **refs.** [2–5]. Moreover, the magnetohydrodynamic fluid flow induced by stretching cylinder along with heat transfer was investigated by Ishak et al. [6]. Mukhopadhyay [7] extended their study by considering slip flow along a cylindrical stretching surface under the effects of magnetic field. The recent developments regarding MHD boundary layer flow brought by cylindrical surface was reported by many researchers [8–15].

The appearing of smog across the mountains is due to stratification phenomena in the atmosphere. The stratification includes both the temperature stratification and the concentration stratification. In boundary layer flows mostly stratification phenomena occurs due to mixing of different fluids having distinct densities, dissolved phases and pressure differences. The transfer of heat from thermal aids like power plants condensers, storage of thermal energy for example solar ponds, heat rejection into system like rivers, seas and lakes are few pertinent applications of stratification individualities. The recent trustful efforts subject to both temperature stratification and concentration stratification can be assessed in Refs. [16–21].

* Corresponding author.
E-mail address: krehman@math.qau.edu.pk (K.U. Rehman).

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Nomenclature

u,v	Velocity components
ν	Kinematic viscosity
n	Power law index
λ	Weissenberg number
$T_w(x)$	Prescribed surface temperature
T_0	Reference temperature
L	Characteristic length
ψ	Stream function
S	Thermal stratification parameter
$\Gamma > 0$	Time dependent material constant
$F'(\eta)$	Velocity of fluid
k	Thermal conductivity
Q_0	Heat generation coefficient
$\theta(\eta)$	Fluid temperature
R	Radius of cylindrical surface
$\frac{Nu_x}{\sqrt{\mathrm{Re}_x}}$	Local Nusselt number
b, c	Positive constant
ζ_1 , ζ_2	Initial guesses
Pr	Prandtl number
$\overleftrightarrow{\mathrm{M}}$	Cauchy stress tensor
$\overrightarrow{\mathrm{V}}$	Velocity vector
μ_0	Zero shear rate viscosity
p	Pressure
$\dot{\gamma}$	Shear rate
x, r	Space variables
ρ	Fluid density
μ	Dynamic viscosity
K	Curvature parameter
$T_\infty(x)$	Variable ambient temperature
U_0	Reference velocity
$F(\eta)$	Dimensionless variable
η	Similarity variable
γ	Magnetic field parameter
c_p	Specific heat at constant pressure
B_0	Uniform magnetic field
$C_F\sqrt{\mathrm{Re}_x}$	Skin friction coefficient
τ_w	Shear stress
$U(x)$	Stretching velocity
Re_x	Local Reynold number
T_0	Reference temperature
$H > 0$	Heat generation parameter
σ	Fluid electrical conductivity
$\overrightarrow{\mathrm{B}}$	Body force
$\overleftrightarrow{\tau}$	Extra stress tensor for tangent hyperbolic fluid
μ_∞	Infinite shear rate viscosity
$\overleftrightarrow{\mathrm{I}}$	Identity tensor
π	Second invariant strain tensor

The study of non-Newtonian fluid models through boundary layer approximations has received considerable attention of researchers having direct or indirect affiliation with the field of fluid science. Such type of fluids are found abundantly in many engineering and industrial processes such as food mixing, plasma, multi grade oils, composite materials, wire drawing, hot rolling, petroleum production, construction of paper a production and many others. To be more specific, all substances that depicts shear thinning characteristics can be explored by means of tangent hyperbolic fluid model. This fluid model is also termed as four constant fluid model. The extra stress tensor for tangent hyperbolic fluid model [22,23] can be written as:

$$\overleftrightarrow{\tau} = [\mu_\infty + (\mu_0 + \mu_\infty)\tanh(\Gamma\dot{\gamma})^n]\dot{\gamma}, \tag{i}$$

where $\dot{\gamma}$ is defined as

$$\dot{\gamma} = \sqrt{\frac{1}{2}\sum_i\sum_j \dot{\gamma}_{ij}\dot{\gamma}_{ji}} = \sqrt{\frac{\pi}{2}}, \quad \textbf{with } \pi = \frac{1}{2}\mathrm{trace}[\mathrm{grad}\overrightarrow{\mathrm{V}} + (\mathrm{grad}\overrightarrow{\mathrm{V}})^t]^2. \tag{ii}$$

To seek shear thinning characteristics by using theoretical grounds we consider $\mu_\infty = 0$ and $\Gamma\dot{\gamma} < 1$. One can obtain the required form as follow

$$\overleftrightarrow{\tau} = \mu_0[(\Gamma\dot{\gamma})^n]\dot{\gamma} = \mu_0[1 + \Gamma\dot{\gamma} - 1]^n\dot{\gamma} = \mu_0[1 + n(\Gamma\dot{\gamma} - 1)]\dot{\gamma}, \tag{iii}$$

here, $\overleftrightarrow{\tau}$, $\overrightarrow{\mathrm{V}}$, μ_∞, μ_0, Γ, n, π and $\dot{\gamma}$, denotes extra stress tensor for tangent hyperbolic fluid, velocity vector, infinite shear rate viscosity, zero shear rate viscosity, time dependent material constant, power law index, second invariant strain tensor and shear rate respectively. The generally accepted flow narrating differential equation can be written as

$$\rho\frac{d\overrightarrow{\mathrm{V}}}{dt} = div\overleftrightarrow{\mathrm{M}} + \rho\overrightarrow{\mathrm{B}}, \text{ with } \overleftrightarrow{\mathrm{M}} = -p\overleftrightarrow{\mathrm{I}} + \overleftrightarrow{\tau}, \tag{iv}$$

where, ρ, $\overleftrightarrow{\mathrm{M}}$, p, $\overleftrightarrow{\mathrm{I}}$ and $\overrightarrow{\mathrm{B}}$ stands for fluid density, Cauchy stress tensor, identity tensor and body force respectively. One can assessed the developments on tangent hyperbolic fluid flow in Refs. [24–30]. To the best of our knowledge the combined effects of magnetic field and heat generation on tangent hyperbolic fluid flow towards cylindrical surface in a thermally stratified media are not discussed until now. Therefore, this study is meant to fill in the gap. The flow field situation in a concerned constrained is translated in terms of partial differential equations. These PDE's are converted into a system of ordinary differential equations. A computational algorithm is executed to yield the numerical solution. The effects logs of an involved parameters on dimensionless quantities (velocity and temperature) are discussed in detail by way of graphs. Further, the skin friction coefficient and heat transfer rate are presented with the help of tables. It is trusted that the given results will serve as a help source for the proceeding analysis. It is important to note that the stratification effects for fluid flow due to rotating disk is not investigated as yet. Therefore, one can extend the idea of present