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Theoretical investigation of shock stand-off distance for non-equilibrium flows over spheres

Hua SHEN [a], Chih-Yung WEN [b,*]

[a] Extreme Computing Research Center, Computer Electrical and Mathematical Science and Engineering Division, King Abdullah University of Science and Technology, Thuwal 23955-6900, Saudi Arabia
[b] Department of Mechanical Engineering, The Hong Kong Polytechnic University, Kowloon 999077, Hong Kong Special Administrative Region

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Abstract We derived a theoretical solution of the shock stand-off distance for a non-equilibrium flow over spheres based on Wen and Hornung's solution and Olivier's solution. Compared with previous approaches, the main advantage of the present approach is allowing an analytic solution without involving any semi-empirical parameter for the whole non-equilibrium flow regimes. The effects of some important physical quantities therefore can be fully revealed via the analytic solution. By combining the current solution with Ideal Dissociating Gas (IDG) model, we investigate the effects of free stream kinetic energy and free stream dissociation level (which can be very different between different facilities) on the shock stand-off distance.

1. Introduction

When a supersonic/hypersonic flow over a blunt body like a sphere, a detached bow shock forms around the body, and the level of the non-equilibrium of the flow is measured by the following dimensionless reaction rate parameter,[1] $\Omega \equiv \left(\frac{\mathrm{d}\alpha}{\mathrm{d}t}\right)_{\mathrm{s}} \frac{D}{2u_\infty}$, where α is the dissociation fraction, D the diameter of the sphere, u the velocity; and the subscripts "∞" and "s" means the corresponding quantities at freestream and immediately behind the shock, respectively. Depending on the value of Ω, the flow can be categorized into nearly frozen flow ($\Omega \ll 1$), nearly equilibrium flow ($\Omega \gg 1$), and non-equilibrium flow (otherwise). The distance between the bow shock and the stagnation point of the nose was referred to as the Shock Stand-off Distance (SSD). The SSD is much smaller than the size of the tested model, and hence experimental measurement admits large errors. Generally speaking, if there is no significant dissociation in the free stream, a larger free stream kinetic energy leads a smaller SSD, due to a higher level of vibrational excitation and chemical dissociation. But an increased SSD is observed in high enthalpy shock tunnels under the same free stream velocity and this phenomenon is attributed to the inevitable free stream dissociation in such facilities.[2,3] In order to understand the physics behind, it is cru-

* Corresponding author.
E-mail address: cywen@polyu.edu.hk (C.-Y. WEN).
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cial to explore the effects of the important flow parameters through theoretical analysis. Olivier et al.[2] first gave an estimation of the effect of free stream dissociation on SSD, but no quantitative solution was provided.

For frozen flows, Lobb[4] performed extensive experiments on the SSD for spheres of various diameters using a schlieren photography technique and derived the following correlation

$$\frac{\Delta}{D} = L\frac{\rho_\infty}{\rho_s}$$

where Δ is the SSD, ρ density, L a constant with a value of 0.41 for spheres. For dissociating flows, the accuracy of Lobb's correlation is significantly degraded.[5,6]

Wen and Hornung[5] proposed an analytic correlation between generalized dimensionless SSD $\left(\tilde{\Delta} \equiv \frac{\Delta}{D} \cdot \frac{\rho_s}{\rho_\infty}\right)$ and the generalized reaction rate parameter $\left(\tilde{\Omega} \equiv \left(\frac{d\rho}{dt}\right)_s \frac{D}{\rho_s u_\infty}\right)$, which comprises two branches, namely a frozen-side and an equilibrium-side. The frozen-side solution is given by

$$\tilde{\Delta} = \frac{1}{\tilde{\Omega}}\left(-1 + \sqrt{1 + 2L\tilde{\Omega}}\right)$$

which implies the SSD is independent of all parameters other than L. Meanwhile, the equilibrium-side solution is given by

$$\tilde{\Delta} = \frac{\rho_s}{\rho_e}\left[L + \frac{1}{2\tilde{\Omega}}\left(\frac{\rho_e}{\rho_s} - 1\right)^2\right]$$

which implies the importance of the density ratio ρ_s/ρ_e (note that the subscript "e" denotes the corresponding quantities at fully equilibrium states). This simple correlation is well validated by experiments,[5,7] CFD results[8,9] and a quasi-one-dimensional model.[10] However, it relies on the semi-empirical parameter L measured by experiments, and therefore cannot completely reveal the embedded physics.

Based on a differential analysis of the governing conservation equations, Olivier[11] proposed the following analytic solution for the SSD in frozen and equilibrium flows:

$$\tilde{\Delta} = \left\{\frac{\rho_s}{\rho_\infty}\sqrt{\frac{1}{4}\left[1 + \left(\frac{\partial \bar{u}}{\partial \phi}\right)_b\right]^2 - \frac{1}{3}\frac{\rho_s}{\rho_b}\cdot\frac{\rho_\infty}{\rho_s}\left[1 + 2\left(\frac{\partial \bar{u}}{\partial \phi}\right)_b\right]} - \frac{1}{2}\left[1 + \left(\frac{\partial \bar{u}}{\partial \phi}\right)_b\right]\frac{\rho_s}{\rho_\infty} + \frac{\rho_s}{\rho_b}\right\}\left[\frac{4}{3} + \frac{2}{3}\left(\frac{\partial \bar{u}}{\partial \phi}\right)_b - 2\frac{\rho_s}{\rho_b}\frac{\rho_\infty}{\rho_s}\right]^{-1}$$

where $\frac{\partial \bar{u}}{\partial \phi}$ is the dimensionless tangential velocity gradient and the subscript "b" represents corresponding values at the stagnation point (body). For frozen air flows, i.e., $\rho_s/\rho_b = 1$ and $\rho_s/\rho_\infty = 6$, the Olivier's analytic solution has a value of $\tilde{\Delta} = 0.4$, and is thus in good agreement with the solution obtained from Lobb's correlation. Significantly, Olivier's model shows that the parameter L is not constant but depends on the gas properties. Nevertheless, since non-equilibrium processes increase the complexity of the conservation equations to such an extent that even for a quasi-one-dimensional approach, analytic solutions cannot be obtained for the whole non-equilibrium flow regimes.[12]

In view of the discussions above, the present study has two aims: (A) to derive a comprehensive analytic solution for the whole non-equilibrium flow regime without using the semi-empirical parameter L; (B) to investigate the effect of two fundamental flow parameters, namely the freestream kinetic energy, and the freestream dissociating level, on the SSD using a simple Ideal Dissociating Gas (IDG) model.[13,14]

2. Analytic solution for shock stand-off distance

Consider the control volume ΔV in the stagnation region between the shock and the body, as shown in Fig. 1. The rate at which mass enters the control volume from the left-hand side is equal to $\rho_\infty u_\infty b$ or $\rho_\infty u_\infty b^2$, depending on whether the flow is two-dimensional or axisymmetric, respectively. Meanwhile, the rate at which mass leaves the control volume through the right-hand side is equal to

$$\int_R^{R+\Delta} \rho u_\tau \, dr \text{ or } 2\int_R^{R+\Delta} \rho u_\tau r \sin\phi \, dr$$

where u_τ is the tangential velocity (i.e., the component of velocity normal to the ray from the center of curvature), R is the radius of the sphere and dr is the differential element of the radius. Consequently, the mass balance is given as

$$\rho_\infty u_\infty b = \int_R^{R+\Delta} \rho u_\tau \, dr \tag{1}$$

and

$$\rho_\infty u_\infty b^2 = 2\int_R^{R+\Delta} \rho u_\tau r \sin\phi \, dr \tag{2}$$

for two-dimensional and axisymmetric flows, respectively. The integral terms in Eqs. (1) and (2) can be approximated using the average value, i.e.,

$$\int_R^{R+\Delta} \rho u_\tau \, dr = \overline{\rho u_\tau}\Delta \tag{3}$$

and

$$\int_R^{R+\Delta} \rho u_\tau r \sin\phi \, dr = \overline{\rho u_\tau}\frac{1}{2}(2R\Delta + \Delta^2)\sin\phi \tag{4}$$

Furthermore, let only the flow region very close to the stagnation streamline be considered. Therefore, the following approximations can be applied:

$$b \approx (R + \Delta)\tan\phi, \quad \sin\phi \approx \tan\phi \approx \phi, \quad u_\tau = \phi\frac{\partial u_\tau}{\partial \phi} \tag{5}$$

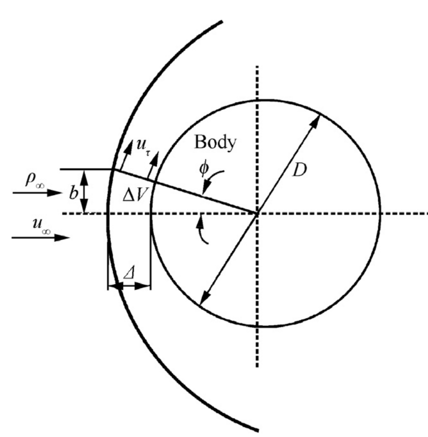

Fig. 1 Schematic of control volume and associated notations.

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