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Aircraft nonlinear stability analysis and multidimensional stability region estimation under icing conditions

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Abstract Icing is one of the crucial factors that could pose great threat to flight safety, and thus research on stability and stability region of aircraft safety under icing conditions is significant for control and flight. Nonlinear dynamical equations and models of aerodynamic coefficients of an aircraft are set up in this paper to study the stability and stability region of the aircraft under an icing condition. Firstly, the equilibrium points of the iced aircraft system are calculated and analyzed based on the theory of differential equation stability. Secondly, according to the correlation theory about equilibrium points and the stability region, this paper estimates the multidimensional stability region of the aircraft, based on which the stability regions before and after icing are compared. Finally, the results are confirmed by the time history analysis. The results can give a reference for stability analysis and envelope protection of the nonlinear system of an iced aircraft.

1. Introduction

Aircraft icing not only increases the weight of an aircraft, but also degrades both the performance and control of the aircraft because icing disrupts the flow of air over the aircraft[1]. Excessive icing accretion can lead to flow separation. It not only causes more resistance and less lift, but also leads to loss of control effectiveness[2], or even aircraft crashes. Twelve percent of flight accidents were caused by icing according to the statistics about flight accidents caused by weather factors between 1999 and 2000, given by America Safety Advisor[3].

On June 3rd 2006, a transport plane crashed in Anhui province. The direct cause is that the aircraft flew through ice clouds many times, and the freezing condition eventually drove the plane out of control. Incidents such as the American Eagle roll upset near Roselawn, Indiana in October 1994 and China Eastern Airlines CRJ-200 plane crash during takeoff at Baotou Inner Mongolia Airport in November 2004 are just two other accidents caused by aircraft icing[4]. The primary cause of these accidents is the effect of ice on the aircraft control effectiveness.

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The effect of ice accretion on aircraft has been observed and investigated systematically since the 1940s[5]. In the early stage, the studies mainly focused on the changes of aerodynamic force and moment of an aircraft under icing conditions[6–8]. Afterwards, the performance, stability, and controllability of an aircraft after icing became the main interests[9–11]. Finally, the methodologies of envelope protection[12,13], anti-icing[14], and de-icing[15] of an aircraft under icing conditions were studied intensively.

The development of safer and more reliable aircraft must include better solutions for flights in icing and other bad weather conditions. Icing accidents can be prevented in two ways: (A) icing conditions can be avoided; (B) aircraft systems can be designed and operated in an ice-tolerant manner. For all kinds of aircraft, ice avoidance is the most desirable goal for enhancing safety and reliability. The main methods are: anti-icing coating, electro-thermal de-icing, electro-impulse de-icing, hot-air anti-icing, and so on.

However, for some aircraft, due to the funding or the weight constraint, ice tolerance will continue to be the preferred method. Sometimes, ice tolerance is necessary in some severe icing conditions. The research of this paper is on the aircraft stability and stability region under icing conditions, especially the multidimensional stability region. The stability region of nonlinear two-dimensional systems has been analyzed in some Refs. 16, 17. However, as we know, the two-dimensional stability region solution is relatively simple and the stability of a two-parameter reaction aircraft system is not comprehensive. If a multidimensional stability boundary of the aircraft system under icing conditions can be obtained through analysis and calculation, we can know the aircraft parameters' tolerance in a certain degree of icing. Then, the aircraft's safety can be judged clearly.

2. Model of aircraft dynamics

The purpose of this work is to analyze the equations of motions for an aircraft under an icing condition. The high angle of attack dynamics of the aircraft are inherently nonlinear. The nonlinearity will be stronger under the icing condition. Therefore, it is essential to study the stability performance of the iced aircraft. The stalling angle and available angle of attack which is the limit angle of attack for safety flight will decrease after icing as shown in Fig. 1. Based on the above reasons, this work concentrates on the high angle of attack dynamics of an aircraft under an icing condition.

2.1. Equations of motion

The equations of motion used in this study assume a rigid aircraft. At the same time, the gyroscopic moment of the engine rotor is negligible, no applied thrust. The equations, consisting of translational equations and rotational equations (18), (19), are written in a principal axis system for analyzing the high angle of attack dynamics as follows:

$$\begin{aligned}\dot{v} = &\left(-g\sin\theta + \frac{QSC_x}{M}\right)\cos\alpha\cos\beta \\ &+ \left(g\cos\theta\sin\phi + \frac{QSC_y}{M}\right)\sin\beta \\ &+ \left(g\cos\theta\sin\phi + \frac{QSC_z}{M}\right)\sin\alpha\cos\beta \end{aligned} \tag{1}$$

$$\dot{\alpha} = q + \frac{-\left(\frac{QS}{Mv}C_x - \frac{g}{v}\sin\theta + r\sin\beta\right)\sin\alpha + \left(\frac{QS}{Mv}C_z + \frac{g}{v}\cos\theta\cos\phi - p\sin\beta\right)\cos\alpha}{\cos\beta} \tag{2}$$

$$\begin{aligned}\dot{\beta} = &-\left[\left(\frac{QS}{Mv}C_x - \frac{g}{v}\sin\theta\right)\sin\beta + r\right]\cos\alpha \\ &+ \left(\frac{QS}{Mv}C_y + \frac{g}{v}\cos\theta\sin\phi\right)\cos\beta \\ &- \left[\left(\frac{QS}{Mv}C_z + \frac{g}{v}\cos\theta\cos\phi\right)\sin\beta - p\right]\sin\alpha \end{aligned} \tag{3}$$

$$\begin{aligned}\dot{p} = &\left[-\left(\frac{I_z - I_y}{I_x} + \frac{I_{xz}^2}{I_x I_z}\right)qr\right. \\ &\left.+\left(1 - \frac{I_y - I_x}{I_z}\right)\frac{I_{xz}}{I_x}pq + \frac{QSb}{I_x}\left(C_l + \frac{I_{xz}}{I_z}C_n\right)\right] \Big/ \left(1 - \frac{I_{xz}^2}{I_x I_z}\right)\end{aligned} \tag{4}$$

$$\dot{q} = \frac{QS\bar{c}}{I_y}C_m + \frac{I_z - I_x}{I_y}pr + \frac{I_{xz}}{I_y}(r^2 - p^2) \tag{5}$$

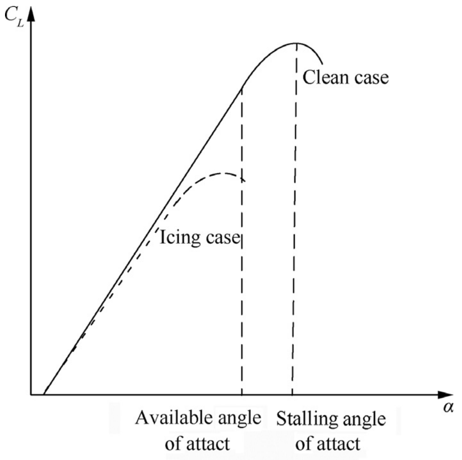

Fig. 1 Sketch map of lift coefficient C_L of aircraft changes in clean case and icing case.

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