Chinese Journal of Aeronautics, (2016), xxx(xx): xxx–xxx

Chinese Society of Aeronautics and Astronautics
& Beihang University
Chinese Journal of Aeronautics
cja@buaa.edu.cn
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FULL LENGTH ARTICLE

Analytical solution methods for eigenbuckling of symmetric cross-ply composite laminates

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Received 4 July 2016; revised 20 July 2016; accepted 5 August 2016

KEYWORDS
Accurate computation;
Analytical solution;
Composite laminates;
Eigenbuckling;
Separation of variables;
State space

Abstract Based on the first-order shear deformation theory, this paper explores the analytical methods for eigenbuckling of symmetric cross-ply rectangular composite laminates with a pair of parallel edges simply supported and the remaining two edges arbitrarily constrained. The main contribution of present paper lies in two aspects: one is to present a simple and effective analytical method, namely, the separation-of-variables method, which can generate the closed-form buckling solutions without any computational difficulty; the other is to incorporate the accurate computation method of exponential matrix into the state space technique to avoid the inevitable numerically ill-conditioned problems reported in several literatures.
The results obtained via both analytical methods are identical, and a good agreement with their counterparts in literature is observed. The separation-of-variables method can generate exact solutions within 1 s, which is impossible if the state space method is employed. Besides, the combination of the accurate computation method of exponential matrix and the state space method greatly improves the computational efficiency and gives correct results compared with the straightforward use of state space method.

1. Introduction

Laminated composite plates are increasingly used in aeronautical, mechanical, civil and marine structures. The buckling analysis of composite plates is essential for reliable and efficient structural design.

Various theories, generally classified into single-layer and multilayer (layer-wise) plate theory, have been proposed to deal with eigenbuckling of composite laminates. Single-layer theories, in which a composite laminate is treated as an equivalent orthotropic and homogeneous single layer, are often used to predict global response characteristics such as maximum

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Peer review under responsibility of Editorial Committee of CJA.
Production and hosting by Elsevier

http://dx.doi.org/10.1016/j.cja.2016.12.027
1000-9361 © 2016 Production and hosting by Elsevier Ltd. on behalf of Chinese Society of Aeronautics and Astronautics.

deflections, natural frequencies and critical buckling loads, while multilayer theories are indispensable to model local effects such as delamination. Thus, for the purpose of present study, only single-layer plate theories are discussed.

The classical laminated plate theory based on the Kirchhoff assumptions was employed for buckling analysis of orthotropic plates by Das,[1] Harik and Ekambaram,[2] Bao et al.,[3] Hwang and Lee,[4] and others. This theory neglects the transverse shear strains, thus overpredicting the buckling loads or even yielding incorrect results especially for composite plates with relatively high ratio of in-plane elastic modulus to transverse shear modulus.

Composite plates are highly flexible in transverse shear; therefore the transverse shear deformation must be taken into account to achieve an accurate representation of laminated plate behavior. Many shear deformation plate theories, which can be extended for analysis of laminated composite plates, are documented in Refs. 5–15.

The first-order shear deformation plate theory (FSDT), commonly known as the Mindlin plate theory (MPT),[7] is representative and one of the most widely used shear deformation theory and provides the best compromise between accuracy and computational efficiency.

Higher-order shear deformation plate theories (HSDT), which involve higher order terms in Taylor's expansion of the displacements in the thickness coordinate, have been proposed by many researchers including Librescu,[9] Levinson,[10] Bhimaraddi and Stevens,[11] Reddy,[12] Ren,[13] Kant and Pandya,[14] Shimpi and Patel[15] and so on. The third-order shear deformation theory by Reddy[12] is representative, in which the shear stress function is parabolic through the thickness and satisfies the stress-free boundary conditions; therefore, shear correction factor is not required.

Considering symmetric cross-ply composite laminates as an equivalent single layer, the standard procedure of determining critical buckling loads is to solve a mathematical eigenvalue problem governed by characteristic differential equations and boundary conditions. However, the exact solutions are limited to a relatively few cases wherein at least two parallel edges are simply supported. For other combinations of edge conditions, approximate procedures such as the Ritz, Galerkin, superposition, finite element and finite difference methods[16–23] should be used. Numerical methods are effective in analyzing plates of arbitrary boundary conditions, physical properties and loading configurations.

Although research regarding the extension of various numerical methods to buckling analysis of composite plates is increasing rapidly, a relative lack of theoretical value and substantial progress exists. In contrast, the development of analytical method is rare, but analytical solution is essential for understanding the physical behavior of plates.

The state space method, which has been widely used in several literatures,[24–36] is a powerful analytical technique to generate Levy-type solutions for eigenbuckling of composite laminates. However, the main drawback of this method is the inevitable computational difficulty caused by the calculation of exponential matrix. As was pointed out by Khdeir[33] that while shearing deformation theories are used, straightforward application of the state-space concept yields numerically ill-conditioned problems as the laminate thickness is reduced. Software with double precision floating point calculation ability may fail to work out the results and the computation time and workload are unsatisfactory. To overcome this computational difficulty, various mathematical methods including the decomposition of matrix and the modified Gram-Schmidt orthonormalization procedure were discussed[34–36] for free vibration and buckling problems, and these methods are not the straightforward use of state space method although they are effective.

In this context, this paper is to develop analytical methods to overcome or even totally avoid computational difficulty for the straightforward use of state space method. The main contribution of present paper lies in two aspects. First, a simple and effective analytical method, which can totally avoid any computational difficulty, is proposed. This method, referred to as separation-of-variables method in this paper, affords explicit eigenvalue equations and closed-form buckling solutions for Levy-type composite plates. Furthermore, it takes less than 1 s to work out the results and the workload is rather small. Second, a combination of the accurate computation method of exponential matrix and the state space concept is introduced to completely solve the inevitable numerically ill-conditioned problems aforementioned, greatly reducing computation time and workload.

Closed-form buckling solutions of symmetric cross-ply rectangular laminates based on FSDT are obtained by both analytical methods and a comparison study is conducted to validate both methods.

2. Governing equations and boundary conditions

A laminated rectangular composite plate of length a, width b and uniform thickness h is considered oriented so that its mid-plane surface contains the x- and y-axis of a Cartesian coordinate system (x, y, z), as shown in Fig. 1.

The displacement field of the first-order shear deformation theory is

$$\begin{cases} u(x,y,z) = u_0(x,y) - z\psi_x(x,y) \\ v(x,y,z) = v_0(x,y) - z\psi_y(x,y) \\ w(x,y,z) = w_0(x,y) \end{cases} \tag{1}$$

where $u_0(x, y)$, $v_0(x, y)$ and $w_0(x, y)$ denote the displacements of a point (x, y) in mid-plane; ψ_x and ψ_y are the rotations of a normal line with respect to y and x coordinates, respectively.

The strain field for the assumed displacement field follows immediately as

$$\begin{cases} \varepsilon_1 = \varepsilon_1^0 - z\kappa_1, \ \ \varepsilon_2 = \varepsilon_2^0 - z\kappa_2 \\ \varepsilon_3 = 0, \ \ \varepsilon_4 = \frac{\partial w}{\partial y} - \psi_y \\ \varepsilon_5 = \frac{\partial w}{\partial x} - \psi_x, \ \ \varepsilon_6 = \varepsilon_6^0 - z\kappa_6 \end{cases} \tag{2}$$

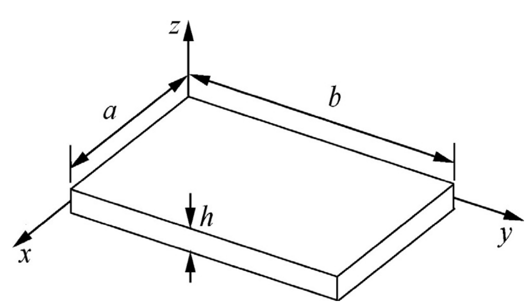

Fig. 1 A rectangular plate and coordinates.