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A new approach on necking constitutive relationships of ductile materials at elevated temperatures

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Abstract A new method is presented to determine the full-range, uniaxial constitutive relationship of materials by tensile tests on funnel specimens with small curvature radius and finite element analysis (FEA). An iteration method using FEA APDL (ANSYS parametric design language) programming has been developed to approach the necking constitutive relationship of materials. Test results from SAE 304 stainless steel at room temperature show that simulated load vs displacement curve, diameter at funnel root vs displacement curve, and funnel deformation contours are close to modeled results. Due to this new method, full-range constitutive relationships and true stress and effective true strain at failure are found for 316L stainless steel, TA17 titanium alloy and A508-III stainless steel at room temperature, and 316L stainless steel at various elevated temperatures.

1. Introduction

The true stress-strain curves of ductile materials before necking initiation can be easily obtained by conventional uniaxial tensile testing using a standard round bar.

$$\begin{cases} \sigma_T = \sigma_E(1+\varepsilon_E) \\ \varepsilon_T = \ln(1+\varepsilon_E) \end{cases} \tag{1}$$

where σ_T is true stress, ε_T is true strain, σ_E is engineering stress, and ε_E is engineering strain. However, after the necking deformation occurs, the acquisition of uniaxial true stress and true strain becomes difficult due to the rapid reduction in local cross section of the straight round bar. Bridgeman[1] made an assumption that the contour of the cross section in the necking deformation zone was circular, and the equivalent strain was uniformly distributed on this section. Therefore, the corrected stress in the necking deformation zone of a round bar could be found as follows:

$$S^* = \frac{S}{(1+4R/d)[\ln(1+d/4R)]} \tag{2}$$

where S is nominal stress and, as shown in Fig. 1, d is the minimum diameter of the cross section in the necking zone, and R is the radius of the necking section.

The Bridgeman correction of stress leads to material curves affected by an error that can be greater than 10% and requires a significant amount of experimental work in order to measure

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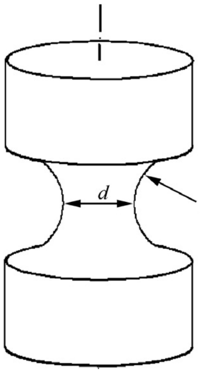

Fig. 1 Necking shape.

the evolving curvature radii of necking profiles at different stages of each tensile test.[2]

To calculate the necking deformation in the work zone of a round bar, Chen[3], Needleman et al.[4,5], and Saje[6] achieved necking simulation of a round bar by finite element analysis (FEA). To induce necking deformation, Chen[3] made an artificial taper on the round bar, Needleman et al.[4,5] used bifurcation criterion and Saje[6] set a rigid restriction at the end of the bar. However, calculation accuracy was limited due to the low level of the technique's use of computers and FEA, and there was no experimental verification for this method. Gurson[7] proposed a void growth model under an axisymmetric stress state to describe the large deformation of materials. Chu and Needleman[8], and Tvergarrd[9] developed a GTN (Gurson-Tvergarrd-Needleman) model by improving Gurson's model. The GTN involves the highly complex determination of nearly 10 parameters and its accuracy cannot be ensured.

Accuracy of simulation results for necking was calculated for the first time by Li.[10] By adjusting main stress and main strain, Norris et al.[11] proposed an preliminary iteration method to obtain the true stress-strain curve after necking. Thereafter, Matic et al.[12–14] proposed a method to evaluate the full-range constitutive relationship of ductile alloys by adjusting the power-law parameters in the constitutive model. However, many materials' constitutive relationships have non-power-law hardening constitutive relationships after necking or even during hardening stages. Zhang et al.[15,16] also used the parameter searching method to acquire full-range constitutive relationships where the load-displacement curve was set to be the target of convergence. Choi[17], Nayebi[18], Cabezas and Celentano[19], and Lee[20] et al. made several attempts to acquire full-range constitutive relationships of ductile materials by different methods, but the resulting strain ranges were limited, and the validity of methods was also questionable. Joun et al.[21,22] completely simulated the necking of a straight round bar without defects using a rigid plastic finite element method. However, the validity and the accuracy of that research still need to be further tested. In recent research reported by Xue et al.[23] full-range constitutive relationships were obtained by the above-mentioned parameter searching method. Yao et al.[24,25] proposed a method to obtain a full-range constitutive relationship by using a standard straight round bar and a funnel-shaped round bar simultaneously, but the dispersion of materials has a remarkable effect on this method that cannot be easily countered.

In this study, the finite element analysis aided testing (FAT) method has been proposed to acquire the full-range constitutive relationships of ductile materials by using a funnel-shaped round bar. This method contains the determination of true stress-strain relationships both before and after necking. The application of the funnel-shaped round bar can directly simulate the necking phenomenon without artificial defects. By directly adjusting the input data of a constitutive relationship in FEA, the full-range true stress-strain curve can be determined if the experimental load-displacement records coincide with the numerical results. Additionally, an optical observation based on a digital image correlation (DIC) technique is employed to verify the validity of the FAT method by checking root diameter variation in a funnel-shaped round bar, outline of the deformed specimen, and strain distribution on the specimen. Based on the proposed FAT method, the full-range constitutive relationships are estimated for SS304 (SAE 304 stainless steel), TA17 titanium alloy and A508-III steel at room temperature, and SS316L (SAE 316L stainless steel) at various elevated temperatures. The critical failure true stress and true strain are also given. Stainless steel is now widely used in the aviation and aerospace industries for things such as engine components and wing parts because of its high strength, elongation and anti-fatigue performance. TA17 titanium alloy is a typical light-weight aerospace alloy applied as the main material for air frames and engines. A508-III steel is a reaction pressure vessel material that has high strength with relatively low hardening. According to analyses of stress and strain distributions on cross sections in the necking zone, the failure mechanisms of ductile materials will been discussed in detail.

2. Research conditions

2.1. Testing system

The uniaxial tensile test system includes the universal test machine material testing system (MTS), room temperature strain extensometer MTS632.12C-21 (25 mm gauge length, 50% measuring range, 5‰ precision) and high temperature strain extensometer MTS632.68F-08 (12 mm gauge length, 20% measuring range, 5‰ precision), centering grips system and VIC-3D (video image correlation-3 dimensional) optical measuring system. The control mode of the tensile test is displacement and the test speed is 0.02 mm/s. The VIC-3D non-contact optical measurement system was used to obtain the diameter at the funnel root d and the deformation contours of the funnel specimen. The impact effect of bias-load was eliminated by the centering grips system. Elastic modulus testing results in four directions of the same specimen showed that the test error does not exceed 0.5% using the centering grips.

2.2. Materials and specimens

The materials to be tested are SS304, A508-III, TA17 and SS316L. SS304, TA17 and SSA508-III were tested at room temperature, and SS316L was tested at room temperature, 300 °C and 500 °C. After solid solution strengthening, the mechanical properties of SS304 and SS316l are quite stable.

Fig. 2 shows dimensions of the straight round bar specimens and funnel-shaped specimens. The radius of the funnel-