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Cascaded lattice Boltzmann method for incompressible thermal flows with heat sources and general thermal boundary conditions

Linlin Fei[a], Kai Hong Luo[a,b,*]

[a] Center for Combustion Energy Key laboratory for Thermal Science and Power Engineering of Ministry of Education, Department of Energy and Power Engineering, Tsinghua University, Beijing 100084, China
[b] Department of Mechanical Engineering, University College London, Torrington Place, London WC1E 7JE, UK

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ABSTRACT

Cascaded or central-moment-based lattice Boltzmann method (CLBM) is a relatively recent development in the LBM community, which has better numerical stability and naturally achieves better Galilean invariance for a specified lattice compared with the classical single-relation-time (SRT) LBM. Recently, CLBM has been extended to simulate thermal flows based on the double-distribution-function (DDF) approach [L. Fei et al., Int. J. Heat Mass Transfer 120, 624 (2018)]. In this work, CLBM is further extended to simulate thermal flows involving complex thermal boundary conditions and/or a heat source. Particularly, a discrete source term in the central-moment space is proposed to include a heat source, and a general bounce-back scheme is employed to implement thermal boundary conditions. The numerical results for several canonical problems are in good agreement with the analytical solutions and/or numerical results in the literature, which verifies the present CLBM implementation for thermal flows.

1. Introduction

In the last three decades or so, the lattice Boltzmann method (LBM), which is a mesoscopic numerical method based on the kinetic theory, has been developed to be a widely used numerical method for solving various fluid flows and heat transfer problems [1–7]. In the LBM, a discretized Boltzmann equation, based on a specific discrete velocity set and designed to reproduce the Navier–Stokes (N-S) equations in the macroscopic limit, is solved for the distribution functions (DFs). Generally, the mesoscopic nature of LBM allows its natural incorporation of microscopic and/or mesoscopic physical phenomena, while the highly efficient algorithm makes it affordable computationally [8,9].

In the extensively used algorithm for LBM, the numerical process can be split into two steps [8–10]: the “collision” step and the “streaming” step. In the collision step, the single-relaxation-time (SRT) or BGK scheme [3] is the most widely used collision operator. In the BGK-LBM, all the distribution functions are relaxed to their local equilibrium states at an identical rate, where the relaxation rate is related to the kinematic viscosity. The BGK–LBM is quite simple to implement and can simulate low Reynolds number flows, but it may have numerical instability at high Reynolds number or low-viscosity flows, as well as inaccuracy of implementing the boundary conditions [11–15]. To overcome these difficulties, the multiple-relaxation-time (MRT) collision operator was proposed in the literature [11,12]. In the MRT–LBM, the DF is transformed into a raw moment space, where different raw moments of the DF can be relaxed at different relaxation rates to the local equilibrium raw moments. Compared with the BGK–LBM, the MRT–LBM can enhance numerical stability by carefully separating the time scales among the kinetic modes [12], as well as improve the numerical accuracy for non-slip boundary conditions by choosing a specified relaxation rate for the energy flux [13–15]. However, Geier et al. argued that the MRT–LBM may also encounter instability for high Reynolds number flows due to the insufficient degree of Galilean invariance and the “cross-talk” effect induced by relaxing the raw moments [16]. By relaxing central moments of the DF in the co-moving frame, a cascaded or central-moment-based operator was proposed in 2006 [16]. In the cascaded LBM, also known as CLBM, the “cross-talk” effect in the MRT–LBM is eliminated naturally, and a higher degree of Galilean invariance for a specified lattice can be preserved readily by matching the higher order central moments of the continuous Maxwell-Boltzmann distribution. By setting the relaxation rates for high-order central moments to be 1, CLBM has been applied to simulate high Reynolds number ($Re = 1,400,000$) turbulent flow using coarse grids without resorting to any tur-

* Corresponding author at: Department of Mechanical Engineering, University College London, Torrington Place, London WC1E 7JE, UK.
E-mail address: K.Luo@ucl.ac.uk (K.H. Luo).

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bulence models [16]. Recently, CLBM has been extended to simulate multiphase flows coupled with the pseudo-potential model [17] by Lycett–Brown and Luo [18]. Compared with the BGK–LBM for multiphase flows, the proposed multiphase CLBM reduces the spurious currents near the phase interface significantly [18], and achieves higher stability range for the Reynolds number [19]. As is known, the basic pseudo-potential model has some drawbacks, such as thermodynamic inconsistency, large spurious currents, and suffers from the problem of the surface tension dependence on the density ratio [9]. More recently, Li et al. proposed an approach of achieving thermodynamic consistency via tuning the mechanical stability condition [20,21], and analyzed the effects of the equation of state on the thermodynamic consistency [22]. Inspired by the methods in [20–22], an improved forcing scheme in the pseudo-potential model was proposed in [23]. By coupling the improved forcing scheme with the cascaded operator, Lycett–Brown and Luo achieved very high parameters in the simulation of binary droplet collisions[24].

More recently, CLBM was first extended to simulate thermal flows by the present authors [25], where a thermal cascaded lattice Boltzmann method (TCLBM) was proposed based on the double-distribution-function (DDF) approach. In our TCLBM, the CLBM is used to simulate the flow field and another total energy BGK–LBM is used for the temperature field, where the two fields are coupled by equation of state for the ideal gas. The proposed TCLBM has been proved to be able to simulate low-Mach compressible thermal flows with commendable stability and accuracy. For incompressible thermal flows without viscous dissipation and pressure work, another CLBM has been constructed on a simpler lattice (D2Q5) to solve the passive-scalar temperature field [26]. Compared with the D2Q5 MRT–LBM for the temperature equation, the proposed D2Q5 CLBM is shown to be better Galilean invariant. Thus a higher characteristic velocity can be adopted for convection heat transfer problems, which decreases the computational load significantly. Although CLBM has been applied to several thermal problems [25,26], less attention has been paid to two important factors: temperature field with a heat source and non-isothermal boundary conditions. In this work, we will present the implementation of a heat source and a general bounce-back scheme for the thermal boundary conditions.

The rest of the paper is structured as follows: In Section 2, a brief introduction for the DDF-based CLBM for incompressible thermal flows is given, followed by the implementation of a heat source and general bounce-back scheme for thermal boundary conditions. Numerical experiments are carried out for several benchmark problems to validate the employed method in Section 3. Finally, a brief summary is given in Section 4.

2. Numerical method

The macroscopic governing equations for incompressible thermal flows can be written as:

$$\nabla \cdot \mathbf{u} = 0, \tag{1a}$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho_0}\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{F}, \tag{1b}$$

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \nabla \cdot (\alpha \nabla \phi). \tag{1c}$$

where $\mathbf{u}$, p, ρ_0, T, ν and α are the velocity, pressure, reference density, temperature, kinematic viscosity, and thermal diffusivity, respectively. The Boussinesq approximation is employed in this work, thus the force field is defined as,

$$\mathbf{F} = -\mathbf{g}\beta(T - T_0) + \mathbf{F}_v, \tag{2}$$

where the gravitational acceleration vector $\mathbf{g}$ points to the negative direction of y-axis, β is the thermal expansion coefficient, T_0 is the reference temperature, and $\mathbf{F}_v$ is an external body force.

2.1. CLBM for the flow field

In the present work, the D2Q9 discrete velocity model [3] is used to simulate two-dimensional problems. As usual, the lattice spacing Δx, time step Δt and lattice speed $c = \Delta x/\Delta t$ are set to be 1. The discrete velocities $\mathbf{e}_i = \left[|e_{ix}\rangle, \left|e_{iy}\right\rangle\right]$ are defined by

$$|e_{ix}\rangle = [0, 1, 0, -1, 0, 1, -1, -1, 1]^\top, \tag{3a}$$

$$\left|e_{iy}\right\rangle = [0, 0, 1, 0, -1, 1, 1, -1, -1]^\top, \tag{3b}$$

where $i = 0, \ldots, 8$, $|\cdot\rangle$ denotes the column vector, and the superscript $\top$ indicates transposition.

For the cascaded collision operator, the collision step is carried out in the central-moment space. The raw moments and central moments of the discrete distribution functions (DFs) f_i are defined as:

$$k_{mn} = \left\langle f_i \middle| e_{ix}^m e_{iy}^n \right\rangle, \tag{4a}$$

$$\tilde{k}_{mn} = \left\langle f_i \middle| (e_{ix} - u_x)^m (e_{iy} - u_y)^n \right\rangle, \tag{4b}$$

and the equilibrium values k_{mn}^{eq} and $\tilde{k}_{mn}^{eq}$ are defined analogously by replacing f_i with the discrete equilibrium distribution functions (EDFs) f_i^{eq}. In this work, a simplified raw-moment set is adopted [26],

$$|\Gamma_i\rangle = [k_{00}, k_{10}, k_{01}, k_{20}, k_{02}, k_{11}, k_{21}, k_{12}, k_{22}]^\top, \tag{5}$$

and so do the central moments $\tilde{\Gamma}_i$. Specifically, the raw moments can be given from f_i through a transformation matrix $\mathbf{M}$ by $|\Gamma_i\rangle = \mathbf{M}|f_i\rangle$, and the central moments shifted from raw moments can be performed through a shift matrix $\mathbf{N}$ by $\left|\tilde{\Gamma}_i\right\rangle = \mathbf{N}|\Gamma_i\rangle$. The formulations for $\mathbf{M}$ and $\mathbf{N}$ can be easily obtained according to the raw-moments set [27]. In the present study, $\mathbf{M}$ and $\mathbf{N}$ are expressed as [26],

$$\mathbf{M} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 1 & 0 & -1 & 1 & 1 & -1 & -1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}, \tag{6a}$$

$$\mathbf{N} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -u_x & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -u_y & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ u_x^2 & -2u_x & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ u_y^2 & 0 & -2u_y & 0 & 1 & 0 & 0 & 0 & 0 \\ u_x u_y & -u_y & -u_x & 0 & 0 & 1 & 0 & 0 & 0 \\ -u_x^2 u_y & 2u_x u_y & u_x^2 & -u_y & 0 & -2u_x & 1 & 0 & 0 \\ -u_y^2 u_x & {u_y}^2 & 2u_x u_y & 0 & -u_x & -2u_y & 0 & 1 & 0 \\ u_x^2 u_y^2 & -2u_x u_y^2 & -2u_y u_x^2 & u_y^2 & u_x^2 & 4u_x u_y & -2u_y & -2u_x & 1 \end{bmatrix}. \tag{6b}$$

The post-collision central moments can be obtained by relaxing each of them to their local equilibrium states independently,

$$\left|\tilde{\Gamma}_i^*\right\rangle = (\mathbf{I} - \mathbf{S})\left|\tilde{\Gamma}_i\right\rangle + \mathbf{S}\left|\tilde{\Gamma}_i^{eq}\right\rangle + (\mathbf{I} - \mathbf{S}/2)|C_i\rangle, \tag{7}$$

where the block-diagonal relation matrix is given by,

$$\mathbf{S} = diag\left([0, 0, 0], \begin{bmatrix} s_+, s_- \\ s_-, s_+ \end{bmatrix}, [s_v, s_3, s_3, s_4]\right), \tag{8}$$