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Axisymmetric thermo-elastic field in an infinite one-dimensional hexagonal quasi-crystal space containing a penny-shaped crack under anti-symmetric uniform heat fluxes

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ABSTRACT

This paper is dedicated to investigating the problem of an infinite one-dimensional hexagonal quasi-crystal medium weakened by a penny-shaped crack and subjected to a pair of anti-symmetric and identical uniform heat fluxes. In view of the anti-symmetry with respect to the crack plane, this problem is formulated by a mixed boundary value problem of the half-space. Based on the general thermo-elastic solution, the mixed boundary value problem is solved by means of the generalized potential theory method. The thermo-elastic field variables in the entire three-dimensional space are explicitly expressed in terms of elementary functions. Some important physical quantities on the crack plane, e.g., temperature, crack slip displacement, shear stress and stress intensity factor, are also presented in closed-forms. Numerical calculations are carried out to validate the present analytical solution and to graphically show the distribution of the thermo-elastic coupling field around the crack. The present solution may be served as a benchmark for the experimental investigations by infrared-thermography technique.

1. Introduction

Quasi-crystals (QCs), which have a quasi-periodicity and a non-crystallographic symmetry, were first discovered by Shechtman [1]. Due to their unusual structure and symmetry, QCs have drawn a great deal of attention of scholars in various fields, such as physics, chemistry, crystallography, and so forth [2]. Unlike that of the conventional crystals, the elasticity of QCs should be composed of a phason field and a phonon field [3,4]. The phonon field is identical to the elastic field of the conventional crystals. The phason field could be understood as a "perturbation" of another period to the basic lattice [2]. Temporarily, the phason field does not have the complete intuition meaning [2]. However, it could be explained by the Landau theory [5], and the physical meaning of phason variables can be briefly explained as a quantity to describe the local rearrangement of atoms in a cell in QCs [2]. A detailed and profound discussion concerning the concept of "phason" is far beyond the scope of the present study, and the interested readers can refer to Bak [3,4], Fan [2], Levine et al. [6], etc.

Up to now, a series of interesting properties of QCs have been observed, such as low adhesion [7], low friction [8], low thermal conductivity [9] and high wear resistance [10,8], which makes QCs enjoy a high potential of practical applications.

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Nomenclature

a	radius of a penny-shaped crack
c_{ij}	elastic constants in the phonon field
F	main potential function
$\overline{F}$	complex conjugate of F
$f_i(z), g_i(z)$	functions used for constructing the thermo-elastic fields
G_1, G_2, G_3	constant coefficients involved in the governing equations
h_0, h_{ij}	constants involved in the potential functions
H_0, H_1, H_2	main potential functions
H_{zi}	stresses in the phason field
I	plane $z = 0$
k_{11}, k_{33}	coefficients of heat conduction
K_1, K_2	elastic constants in the phason field
K_{II}, K_{III}	stress intensity factors for the mode II and III crack problems
l_1, l_2, l_{14}, l_{24}	characteristic lengths
M	arbitrary point in the three-dimensional space
n_i	constant coefficients related to the material constants and involved in the characteristic equation
N, N_0	arbitrary points in the region S
q_x, q_y, q_z	heat fluxes in the $x-$, $y-$ and z-directions, respectively
q_0	uniform heat flux load
$R(\cdot, \cdot)$	distance between two points
R_1, R_2, R_3	phonon-phason coupling elastic constants
s_i	eigenvalues of the material
S	region occupied by the crack
$\overline{S}$	intact region in the plane $z = 0$
T, T_0, T_1	temperature increments
u_i	phonon displacements
u_r^e	radial phonon displacement without the phason effect
U	combination of phonon displacements, $U = u_x + \mathrm{i}u_y$
w_z	phason displacement in the $z-$direction
α_{ij}, γ_{ij}	constants related to the material constants
β_1, β_3	thermal modules
δ_{ij}	Kronecker delta
Δ	planar Laplacian operator $\partial^2/\partial x^2 + \partial^2/\partial y^2$
Λ	differential operator $\partial/\partial x + \mathrm{i}\partial/\partial y$
$\overline{\Lambda}$	complex conjugate of the differential operator Λ
σ_{ij}	phonon stresses
σ_1, σ_2	combinations of phonon stresses
σ_{zr}^e	shear phonon stress without the phason effect
τ_g	equivalent shear load
τ_{z1}	combinations of shear phonon stresses, $\tau_{z1} = \sigma_{zx} + \mathrm{i}\sigma_{zy}$
τ_{z2}	combinations of shear phason stresses, $\tau_{z2} = H_{zx} + \mathrm{i}H_{zy}$
Ψ_i	potential functions
IRT	infrared-thermography
QC	quasi-crystal
SIF	stress intensity factor

For example, considering QC structures could exhibit more absorption peak in long wavelength range, Varadarajan and Shekar [11] have fabricated photonic QCs in photoresist to act as a component of the substrate/or back reflector for the solar cell. However, some experimental observations have indicated that QCs are brittle at room temperature [12,2]. On the basis of past experience of conventional material, the existence of macroscopic cracks or flaws are the basic cause of the failure of brittle materials. Therefore, the research on crack problems in QCs becomes extremely significant and necessary.

In the past two decades, a variety of theoretical studies on crack problems in QCs have been reported in literature [2]. For instance, making use of the Fourier transform technique in conjunction with a general solution, Li et al. [13] investigated a Griffith crack problem in a decagonal QC solid, and obtained the entire phonon-phason stress field and the asymptotic field near the crack tip. By the method of Hankel transforms, Peng and Fan [14] studied the problem of a flat circular crack under uniform phonon and phason loads in one-dimensional (1D) hexagonal QCs, and derived the stresses and displacements in the integral forms involving Bessel functions. The problems of anti-plane cracks in 1D hexagonal QCs have been researched by Guo and his coauthors [15–17] and Shi [18], and the stress intensity factors (SIF) for those mode III crack problems were

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