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Explicit solutions of an elliptic hole or a crack problem in thermoelectric materials

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ABSTRACT

The generalized two-dimensional problem of an elliptic hole or a crack in a thermoelectric material subjected to uniform electric current density and energy flux at infinity is investigated based on the complex variable method and the conformal mapping technique. Firstly, the exact solutions of electric potential, temperature and stress fields are presented with the assumption that the surfaces of the elliptic hole are electrically and thermally insulated. It is shown that both the concentration factors of electric current density and stress at hole rim increase as the value of major to minor axis ratio of the elliptic hole increases. Then, explicit solutions are also obtained in closed-form when the elliptic hole degenerates into a crack. It is found that all fields exhibit an inverse square-root singularity at the crack tip, and electric current density and stress intensity factors are defined according to the traditional way. The results show that the mode-I and mode-II stress intensity factors are dependent on the applied electric current density and energy flux, respectively. Furthermore, the mode-I stress intensity factor induced by Joule heating effect has a non-linear relationship with the remote electric loads.

1. Introduction

Thermoelectric materials, which interconvert heat energy and electrical energy on a solid state basis and thus provide a method of power generation and cooling without moving parts and refrigerant [1–3]. As environment-friendly functional materials, they have extensively potential applications in modern optical telecommunications, carbon reduction, bioanalytical instrumentation, satellite and spacecraft industries, high performance liquid chromatography, freezers and waste heat recovery system [4,5]. Design of thermoelectric devices is calling for a better understanding of the responses of these materials subjected to applied electric potential and temperature loads. The nonlinear finite element analysis of thermoelectric materials shows that effect of using material properties depending on the temperature is small for the small range of temperatures [6], however, it becomes remarkable at large temperature differences and high currents [7]. A transient simulation model was presented in order to investigate the influence of rectangular pulsed heat power on the thermal and electrical performances of power generator [8]. These works are very useful for the study and design of thermoelectric semiconductors since the miniaturization temperature, electric fields interaction and residual stress are becoming very important issues.

Severe internal thermal stresses may arise in thermoelectric materials due to thermal shock, thermal cycles and mismatch of the thermo-electro-elastic properties between different materials under in-service conditions [9,10]. It is found that

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Nomenclature

$\mathbf{j}_e$	electric current density vector
$\mathbf{q}$	heat flux vector
$\mathbf{j}_u$	energy flux vector
γ	electric conductivity
k	thermal conductivity
ε	Seebeck coefficient
a	semi-major axis of an elliptic hole
b	semi-minor axis of an elliptic hole
s	arc-length
V	electric potential
T	temperature
μ	shear modulus
ν	Poisson's ratio
α	linear thermal expansion
σ_{ij}	stress
u_i	elastic displacement
K_e	electric current density intensity factor
K_I	mode-I stress intensity factor
K_{II}	mode-II stress intensity factor
G	energy release rate
J	J-integral

thermal stress has a decisive effect on the life expectation of the thermoelectric device under high heat flux imposing upon the hot end [11]. On the other hand, one great problem of most thermoelectric materials is their inherent brittleness and low fracture toughness [12]. Defects/imperfections produced inside the thermoelectric structures and composites during fabrication and/or operational loads, such as holes and cracks, can cause geometric, electric potential and temperature discontinuities across the defect surfaces, and thermoelectric fields and stress concentrations, and then lead either to mechanical failure or electric breakdown. The assessment of these defects plays an important role for the energy conversion efficiency, strength and reliability of thermoelectric components. Therefore, much attention has been devoted to defect problems of thermoelectric materials. The main focus of most current efforts appears to be on experimental research. For example, the slow crack propagation behavior and room-temperature mechanical properties of Mg_2Si-based thermoelectric materials was studied by Schmidt et al. [13]. Fracture toughness of undoped Co_4Sb_{12} and indium doped $In_{0.1}Co_4Sb_{12}$ skutterudites was measured by Vickers indent crack opening displacement, Vickers indentation fracture and single edge vee-notched bend in 4-point flexure methods [14]. Based on a linear simplification of the governing equations, an analytic solution for the thermoelectric materials containing a general 2D crack problem was first derived theoretically by Zhang and Wang [15]. Hereafter, the effective material properties for a thermoelectric composite material with elliptic inclusions were investigated by using conformal mapping function and Mori–Tanaka mean-field method [16]. In engineering practice, the thermoelectric material governing equations combining Seebeck effect, Peltier effect and Fourier's law are intrinsically nonlinear. Naturally, understanding of nonlinear fracture behavior of thermoelectric materials is important [17]. It is well known that an elliptic hole problem is the basis of fracture mechanics analysis for each functional material [18–23]. All these works have led to the development of a powerful continuum framework to study thermo-electro-mechanical effects in hole or crack. Some insights from these literatures are used in this paper. However, to the authors' knowledge, study of the elliptic hole problem in a thermoelectric material has yet not been reached due to the complicated nonlinear governing equations.

In the present work, the elliptic hole or a crack problem in a nonlinear coupled thermoelectric material subjected to remote electric current density and energy flux is theoretically investigated. Thermally and electrically insulated boundary conditions are adopted. The paper is organized as follows. Firstly, basic theory and the general solutions of the thermoelectric material are outlined based on the complex variable technique. Next, solutions of the electric potential, temperature and stress fields for an elliptic hole in a thermoelectric material are derived in closed-form. Then, when the elliptic hole degenerates into a through crack, exact and explicit expressions for the fields intensity factors at the crack tip are given. Finally, the J-integral fracture criteria is introduced and some concluding remarks are made.

2. Basic equations for thermoelectric materials

2.1. Governing equations

Consider an infinite thermoelectric material in which all fields are assumed to depend only on the in-plane coordinates x and y. In the stationary case when no free electric charge and heat source exists, the governing field equations for a homogeneous thermoelectric material can be written as [6],