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Stress intensity factor and energy release rate of externally pressurized thick cross-ply (very) long cylindrical shells with low-hardening transverse shear modulus nonlinearity

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ABSTRACT

Combined effects of modal imperfections, transverse shear/normal deformation along with the nonlinear hypo-elastic transverse shear (G_{TT}) material property on the emergence of interlaminar shear crippling type instability modes, related to the localization (onset of deformation softening), delocalization (onset of deformation hardening) and propagation of mode II compression fracture/damage, in thick imperfect cross-ply very long cylindrical shells under applied hydrostatic pressure, are investigated. The primary accomplishment is the (hitherto-unavailable) computation of the layer-wise mode II fracture toughness, fracture energy and kink crack band-width, under hydrostatic compression, from a nonlinear Finite Element Analysis (FEA), using Maxwell's construction and Griffith's energy balance approach.

1. Introduction

Although localization was first discovered by Anderson [1] as a quantum mechanical phenomenon, localization and delocalization are multi-physics phenomena that transcend atomic scales. In Anderson localization model for opto-electronics [1] an electron is permitted to hop only to the nearest neighbor sites [1,2]. Almost all eigenstates are, according to this model (involving uncorrelated disorder), are known to be localized, and the quantum diffusion of an initially localized wavepacket is suppressed at the localization length [3]. de Moura and Lyra [3] have investigated one-dimensional Anderson models with long-range correlated diagonal disorder. Li et al. [4] have theoretically studied the behavior of electronic states in disordered systems subject to a strong nonresonant optical field, and have concluded that when the optical intensity reaches a critical value, which is dependent on the site-to-site coupling and the standard deviation of the random energy in the disordered system, a sharp transition from localized to extended electronic states takes place. Albrecht and Wimberger [5] have investigated two mechanisms that are responsible for delocalization, namely a correlated disorder potential and mutual interaction between two bosons. Stockman et al. [6] have shown that the eigenmodes (surface plasmons) of disordered nanosystems (modeled as random planar nano-composites) do not always undergo Anderson localization alone, but can display properties of both localized and delocalized states simultaneously.

In the continuum scale, Hunt and Lucena Neto [7] have investigated the influence of localized imperfections on the elastic buckling of a long cylindrical shell (with large Batdorf parameter) under axial compression by using a double scale analysis

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Nomenclature

d_f	fiber diameter
E_{LL}, E_{TT}, ν_{LT}	longitudinal and transverse Young's moduli, and major Poisson's ratio, respectively, of a unidirectional lamina
G_{LT}, G_{TT}	longitudinal and initial transverse shear moduli, respectively
h	thickness of a laminated shell
h_j, h_{j+1}	distance of the top and bottom surface of a sub-lamina from the bottom surface of the corresponding lamina
m	ratio of reference yielding shear stress to transverse shear modulus, G_{TT}, in the Ramberg–Osgood representation
N	total number of laminas
N_s	total number of sub-laminas
n	inverse strain hardening parameter in the Ramberg–Osgood representation
p	uniform hydrostatic pressure
p_{cr}	classical buckling pressure of a very long cylindrical shell (plane strain ring)
R_i	inner radius of a long perfect cylindrical shell (plane strain ring)
$r(\theta)$	radial coordinate of the innermost (bottom) surface of an undeformed ring with modal imperfection
$S_{\alpha\gamma}^{(k)}$	elastic compliances of a lamina
$\{_0V\}$	global displacement vector in FEA
W_0	amplitude of modal imperfection of a plane strain ring
x, β, z	coordinates of a point inside a layer
ε_f, ε_e	force and energy convergence criteria, respectively
θ	angle measured from the global x_3 axis
$\rho(k)$	radius of curvature of the inner surface of the kth layer of an imperfect plane strain ring
Σ_{ij}	component of the incremental compliance matrix of a lamina
ζ	radial distance of a point inside a laminated cylindrical shell/ring measured from its innermost or bottom surface

including interaction modes. An investigation on the effect of localized imperfection must account for a large number of nearly coincident buckling modes and their nonlinear interactions, which can be accomplished by using amplitude equations of the Ginsburg–Landau type [8]. Localized buckling of a modally imperfect ring with bilinear hypoelastic material property has been analyzed by Kim and Chaudhuri [9]. The post-buckling behavior of a thin [90/0/90] cylindrical shell of finite length, and weakened by the presence of a localized imperfection, with surface-parallel shear modulus nonlinearity, is characterized by a limit or localization point beyond which the equilibrium path is unstable, although the same cylinder without localized imperfection and/or material nonlinearity displays stable post-buckling behaviors [10].

In what follows, occurrence of localization/delocalization in thick imperfect cross-ply very long cylindrical shells (plane strain rings), under applied hydrostatic pressure, is investigated. Most carbon fiber reinforced plastic (CFRP) thick cylindrical shells tested at Naval Surface Warfare Center (NSWC), Carderock, MD have failed at much lower pressures than expected [11–13]. Failure has usually been instantaneous and catastrophic, which has also been observed by Hahn and Sohi [14], Abdallah et al. [15], and others. Figs. 1 and 2 show typical failures observed in thick ($R_i/h \approx 6$) CFRP cylinders tested at NSWC, Carderock [11–13]. Observations made from the fractured portion and detailed theoretical analysis indicate that the failure may have initiated at a stress concentration site such as initial fiber waviness or misalignment, shown in Figs. 3 and 4, and associated resin-rich area. As seen in Figs. 1 and 2, the failure creates a longitudinal crack through entire length of the cylinder at kink boundary angle, $\bar{\beta}$, in the range of 28–35° through the cross section. The fibers are broken into quite short lengths along almost the entire cylinder length within the kink band; failed cross-ply ring specimens, tested by Abdallah et al. [15] are shown in Figs. 9 and 10 of Chaudhuri [17]. The collapse pressure and apparent failure mode are about the same for all the carbon-composite thick cylindrical shell models tested at NSWC [11–13]. It has been concluded that initial fiber misalignment, ultimate shear strain of fibers and the two transverse shear moduli of a laminate are the key parameters limiting the compressive strength [16]. An analytical/experimental effort at improving the compressive strength/toughness of the CFRP material by using a hybrid carbon/glass commingling concept has been reported by Chaudhuri and Garala [17]. A Griffith type of fracture criterion for the stability of damage/crack growth, based on the principle of energy balance, is introduced to derive the then-unknown concept of kink toughness (i.e., resistance to kink band propagation) [17].

Propagation of kink band appears to be due to unstable (mode II) crack growth in the neighboring fibers along the kink band angle, $\bar{\beta}$, triggered by the energy released by the kinking or fracture of a defective (e.g., wavy or misaligned) fiber bundle. Chaudhuri et al. [18] and Chaudhuri [19] have investigated kink propagation in glass fiber reinforced unidirectional composites, utilizing eigenfunction expansion techniques [20–23] applied to fully bonded fiber–matrix kink-wedges. A three-dimensional eigenfunction expansion technique, based in part on separation of the thickness-variable and partly utilizing a modified Frobenius type series expansion [24–30] in conjunction with what is known as "Eshelby-Stroh formalism" has been used to compute the local stress singularity, in the vicinity of a kinked-fiber/matrix trimaterial junction front, representing a measure of the degree of inherent flaw sensitivity of unidirectional CFRP under compression. This process is further facilitated by the "yielding" and subsequent "plastic" deformation of the supporting matrix material, being investigated here.