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The sliding contact problem for an orthotropic coating bonded to an isotropic substrate

Y. Alinia[a,*], M. Hosseini-nasab[a], M.A. Güler[b]

[a] Department of Mechanical Engineering, Hakim Sabzevari University, Sabzevar, Iran
[b] Department of Mechanical Engineering, TOBB University of Economics and Technology, Ankara, 06560, Turkey

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ABSTRACT

We consider the sliding contact problem of an orthotropic coating/substrate system. The coating/substrate system is pressed by a rigid flat or cylindrical stamp. For the orthotropic coating, the principal material directions are assumed to be parallel and perpendicular to the contact surface. The governing integral equations corresponding to the mentioned contact problem are extracted by means of the Fourier transform technique. Later, the numerical solution of the singular integral equations is provided by applying the Gauss-Chebyshev integration method. The main goal of this study is to obtain analytical benchmark solutions in order to examine the effect of material orthotropy parameters, relative stiffness, the coefficient of friction and the coating thickness on the stress distribution at the surface of the orthotropic coating. The behavior of the surface in-plane stress intensity factor is analyzed as well. For a constant value of the applied load, the results indicate that the stiffness ratio and the shear parameter have a more pronounced effect on the surface stress components than the effective Poisson's ratio. Also, the stress intensity factor at sharp edges of the flat punch decreases as the coating softens with respect to the substrate and/or the coating thickness decreases.

1. Introduction

The contact mechanics is the backbone of the solid mechanics since the loads between mechanical components are transferred through mechanical contact. Hence for the contacting bodies, the assessment of the contact stresses is so important because of high stress gradients which may occur within the contact area. Subsequently, these local stress concentrations can lead to surface damages, wear and/or crack nucleation. Thus, design and analysis of protective coatings against severe contact stresses is of special interest.

On the other hand, modern materials such as functionally graded materials (FGMs) and composite materials offer superior tribological performance under the contact damages (Jørgensen et al., 1998; Enomoto and Yamamoto, 1998; Suresh et al., 1999; Suresh, 2001; Stewart et al., 2005). Therefore, the contact behavior of the mentioned materials must be well understood. It is clear that the governing equations of these materials become more complicated as a result of the complexity in their micro-structure. Accordingly, numerous attempts have been conducted to disclose the contact behaviors of non-homogeneous and anisotropic materials (Jørgensen, 1994; Jørgensen et al., 1998; Guler and Erdogan, 2004; Zhang et al., 2007).

The contact mechanics analysis of the anisotropic materials has been extensively studied within the literature. The early attempts to the analytical solution for the contact problem of anisotropic materials are available in the textbook written by Galin et al. (2008), Lekhnitskiĭ (1963) and Gladwell (1980). Klintworth and Stronge (1990) employed the complex potential approach for the contact behavior of an anisotropic half-space subjected to rigid punch indentation. They extracted the stress field induced by the normal, the tangential and the moment loading conditions. Kuo and Keer (1992) investigated the spherical indentation of a transversely isotropic multi-layered/half-space system by means of the displacement potential technique. Fan and Keer (1994) utilized both the Eshelby and the Stroh's formalism to find a general solution for the contact problem of an anisotropic half-space. Fan and Hwu (1996) provided an exact solution to the contact problem of an anisotropic half-plane using the combined Stroh's formalism and the analytical continuation method. Wienecke (1999) extracted the Green's function for a transversely isotropic piezoelectric half-space corresponding to both axi-symmetric and transverse loading conditions. The closed form solution to the indention of anisotropic materials under the action of collinear punches was addressed by Zhou and Kim (2014). Mokhtari et al. (2016) considered the frictional contact problem between a rough rigid surface and a transversely isotropic viscoelastic half space. Axisymmetric indentation of a transversely isotropic functionally

* Corresponding author.
E-mail addresses: yaalinia@gmail.com, y.alinia@hsu.ac.ir (Y. Alinia).

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graded coating bonded to a homogeneous half-plane was studied by Vasiliev et al. (2017) with aid of the Hankel transform technique.

Accordingly, the contact mechanics of orthotropic material has been considered in several studies. Ning et al. (2003) studied the effect of the fiber orientation on the contact characteristics of the fiber reinforced polymer composites using the solution method developed by Hwu and Fan (1998). Fukumasu and Souza (2006) conducted some finite element simulations to study the elasto-plastic behavior of a thin film/orthotropic substrate system under the normal indentation. Zhou and Lee (2012) considered the frictionless contact problem of an orthotropic piezoelectric medium. They have utilized the Galilean transformation to incorporate the inertial terms due to the moving punch. Green's function for the orthotropic coating substrate system under the surface line load was given by Hou et al. (2015) with the virtue of the differential operator theory. Guler (2014) provided some closed form solutions to the sliding contact problem of an orthotropic half-space using the singular integral equation technique. Kucuksucu et al. (2015) analyzed the two-dimensional contact problem of a functionally graded orthotropic half-plane. They extracted the strength of contact pressure singularity at the sharp corners of the rigid punches. Sarikaya and Dag (2016) formulated the surface crack problem of an orthotropic medium under the sliding contact condition to explore the effect of material orthotropy on the mixed-mode stress intensity factors. Recently, Güler et al. (2017) used both analytical and finite element methods to simulate the sliding contact problem of an orthotropic functionally graded medium. They investigated the effect of both material inhomogeneity and orthotropy parameters on the VonMises stress distribution.

Generally, the contact problems are simulated under the assumption of the material isotropy. However, most of the new engineered materials such as composites and functionally graded materials (FGMs) exhibit anisotropy. Hence, it will be more appropriate to analyze the behavior of the advanced materials by incorporating their realistic anisotropy. Nowadays, FGMs are widely used in the practical application such as bone remodeling and dental restoration (see for example: Hedia et al. (2006); Huang et al. (2007); Niu et al. (2009); Fouad (2011); Du et al. (2013). On the other hand, FGMs generally exhibit orthotropic behavior as a result of their oriented microstructure. This behavior is a consequence of the processing technique. For example, plasma sprayed coatings have a lamellar microstructure with weak cleavage planes parallel to the boundary (Sampath et al., 1995; Sevostianov and Kachanov, 2001). On the other hand, coatings processed by the electron beam physical vapor deposition (EB-PVD) technique have a columnar structure with weak cleavage planes perpendicular the free surface (Kaysser and Ilschner, 1995; Schulz and Schmücker, 2000). Also, fiber-reinforced composites with a variable fiber volume fraction can be considered as orthotropic functionally graded materials (Benatta et al., 2008).

The current study formulates the general contact problem of an orthotropic coating/substrate system under the analytical framework which has been not considered to best of the author knowledge. Fig. 1a illustrate a short abstract of the literature for the related problem conducted in this area. The contact problem for the isotropic coating-substrate system has been well studied (see for example King and O'sullivan (1987)). On the other hand the contact problem for an orthotropic medium has been analyzed by Guler (2014). The only study for the orthotropic coating was addressed by Hou et al. (2015) who considered surface point load to arrive at Green's functions for the related problem. This study aims to simulate the general contact problem of an orthotropic coating-substrate system.

The derived governing singular integral equations can be applied to any type of contact problems (for example the normal contact, the partial slip contact, the fretting contact, the sliding contact and the rolling contact problems). In this paper, the numerical solution is provided for the sliding contact problem type. Finally, a comprehensive sensitivity analyzed is conducted to illustrate the effect of material orthotropy on the contact stresses and the stress intensity factors. The results indicate that the contact behavior of the orthotropic materials significantly differs from their isotropic counterpart. Also, the load bearing capacity of the coated system can be improved by appropriate adjustment of the material orthotropy parameters.

2. Formulation of the problem

Consider the 2-D plane elasticity contact problem of an orthotropic coating bonded to an isotropic substrate given in Fig. 1b. The medium (1) is an orthotropic coating with a thickness of h and the medium (2) is a homogeneous isotropic substrate. The coordinate system (x_1, x_2) denotes the principal axes of material orthotropy for the coating which are assumed to be parallel and perpendicular to the contact surface. The rigid punch is under the action of the normal load, P, and the tangential load, Q. Following Krenk (1979), the stress-strain relation for an orthotropic medium can expressed as:

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ 2\varepsilon_{12} \end{Bmatrix} = \frac{1}{E}\begin{bmatrix} \delta^{-2} & -\nu & 0 \\ -\nu & \delta^2 & 0 \\ 0 & 0 & 2(\kappa+\nu) \end{bmatrix}\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix}, \tag{1}$$

where the material constants E, ν, δ and κ are known as the effective stiffness, the effective Poisson's ratio, the stiffness ratio and the shear parameter, respectively. The relation between the mentioned constants and the engineering constants of the orthotropic material are given in Appendix A. Also, the shear modulus and the Poisson's ratio of the isotropic substrate are represented by μ_0 and ν_0, respectively.

In order to facilitate the mathematical manipulation, the following coordinate transformation is introduced for the orthotropic coating (Güler et al., 2017):

$$x = \frac{x_1}{\sqrt{\delta}}, \tag{2a}$$

$$y = x_2\sqrt{\delta}, \tag{2b}$$

$$\tilde{u}_1(x, y) = \sqrt{\delta}\, u_1(x_1, x_2), \tag{2c}$$

$$\tilde{v}_1(x, y) = \frac{1}{\sqrt{\delta}} v_1(x_1, x_2), \tag{2d}$$

$$\sigma_{1xx}(x, y) = \sigma_{1x_1x_1}(x_1, x_2)/\delta, \tag{3a}$$

$$\sigma_{1yy}(x, y) = \delta\, \sigma_{1x_2x_2}(x_1, x_2), \tag{3b}$$

$$\sigma_{1xy}(x, y) = \sigma_{1x_1x_2}(x_1, x_2), \tag{3c}$$

where $u_1(x_1, x_2)$ and $v_1(x_1, x_2)$ denote the displacement components within the coating along x_1 and x_2 directions, respectively. Now for the orthotropic coating, the Hooke's law takes the following form:

$$\sigma_{1xx}(x, y) = \frac{E}{1-\nu^2}\left[\frac{\partial}{\partial x}\tilde{u}_1(x, y) + \nu\frac{\partial}{\partial y}\tilde{v}_1(x, y)\right], \tag{4a}$$

$$\sigma_{1yy}(x, y) = \frac{E}{1-\nu^2}\left[\nu\frac{\partial}{\partial x}\tilde{u}_1(x, y) + \frac{\partial}{\partial y}\tilde{v}_1(x, y)\right], \tag{4b}$$

$$\sigma_{1xy}(x, y) = \frac{E}{2(\kappa+\nu)}\left[\frac{\partial}{\partial y}\tilde{u}_1(x, y) + \frac{\partial}{\partial x}\tilde{v}_1(x, y)\right]. \tag{4c}$$

Similarly for the isotropic substrate, one may have:

$$\sigma_{2x_1x_1}(x_1, x_2) = \frac{\mu_0}{\kappa_0 - 1}\left[(\kappa_0+1)\frac{\partial u_2}{\partial x_1} + (3-\kappa_0)\frac{\partial v_2}{\partial x_2}\right], \tag{5a}$$

$$\sigma_{2x_2x_2}(x_1, x_2) = \frac{\mu_0}{\kappa_0 - 1}\left[(3-\kappa_0)\frac{\partial u_2}{\partial x_1} + (\kappa_0+1)\frac{\partial v_2}{\partial x_2}\right], \tag{5b}$$

$$\sigma_{2x_1x_2}(x_1, x_2) = \mu_0\left[\frac{\partial u_2}{\partial x_2} + \frac{\partial v_2}{\partial x_1}\right], \tag{5c}$$