International Journal of Mechanical Sciences 137 (2018) 121–132

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Dynamics comparison of rotating flexible annular disk under different edge boundary conditions

Yong-Chen Pei*, Jun-Heng Wang, Fan Yang

School of Mechanical Science and Engineering, Jilin University, Nanling Campus, Changchun, 130025, People's Republic of China

ARTICLE INFO

Keywords:
Rotating disk
Boundary condition
Natural frequency
Dynamic stability
Critical speed

ABSTRACT

Rotating annular disk has many practical applications, and its inner and outer edge boundary conditions must be selected to apply the rotating disk with different purpose. There are several potential inner and outer edge boundary conditions that can be selected for the rotating disk, but the corresponding dynamics characteristics become a problem that makes the disk work stably and efficiently. In this paper, 25 full combinations of edge boundary conditions are considered for the rotating flexible annular disk. Then their natural frequency, dynamic stability, critical and limit speeds, and steady state response amplitude under initial transversal runout are discussed and compared systematically. This paper is meaningful and beneficial to select the most available and suitable scheme of edge boundary conditions in the rotating disk application.

1. Introduction

The rotating annular disk has many practical applications, such as general saw blades for material cutting and truncating, CD/DVD and hard disk drives for data storage, inner diameter saw blades for crystal slicing, and elastic connection disk for torque transmission. The inner and outer edge boundary conditions of rotating disk are selected to apply the rotating disk with different purpose.

For general saw blades, CD/DVD and hard disk drives, and atomizing disks, the rotating disk is fixed at its inner edge and free at its outer edge. Chonan [1] presented a simple arithmetic equation to determinate the critical speed of this kind of disk. It was proved that the natural vibration frequencies with one nodal diameter are under the restriction of the rotation frequency for the axisymmetric disk by Renshaw et al. [2]. D'Angelo and Mote [3] evaluated the prediction validity of the natural frequencies for the rotating disk with the inner boundary clamped by collars. Lee et al. [4] experimentally studied the critical and flutter speeds of ASMO and CD/DVD disks. In Koo [5], it was indicated that the circumferentially-reinforced CD disks are more effective than the radially-reinforced CD disks to increase critical speed. Young and Lin [6] analyzed the elastic stability of a rotating flexible disk under a stationary oscillating unit. Ouyang [7] numerically studied the non-stationary vibration of the atomizing disks with the same inner and outer boundaries. Pei and Tan [8] presented the significance of the modal interactions in dynamic analysis of the rotating disk with a clamped inner boundary. Khorasany and Hutton [9] investigated the transverse linear vibration of a flexible annular disk rotating at a constant angular speed and free to have rigid body transverse translation. For the functionally graded rotating disks, the transverse vibrations of isotropic linear elastic rotating solid disk (circular plate) was studied by Güven and Çelik [10], and the in-plane stress analysis was carried out for a non-uniform thickness disk rotating at variable angular velocity by Zheng et al. [11]. With a circumferential open crack, Bahaloo et al. [12] investigated the transverse vibration and stability of a rotating annular disk under inner fixed and outer free edge boundary conditions. Oppositely, in the case of the inner diameter saw blades, the rotating disk is free at its inner edge and fixed at its outer edge. Jiang et al. [13,14] experimentally developed an online system to monitor the vibration states of ID (inner diameter) blade slicer cutting a silicon ingot. Ishikawa et al. [15] presented a technique to improve the ID blade slicing efficiency and accuracy by utilizing the elliptical vibration. With considering the reaction of ingot and the negative air pressure between the blade and the ingot, Chonan et al. [16] analyzed the vibration stability of ID blade cutting a crystal ingot. For the elastic connection disk between the vehicle engine and the hydraulic torque converter, the rotating disk is fixed at both its inner and outer edges to transmit a torque. Wu [17] presented a numerical model on torsional vibration of an elastic connection disk. Maretic et al. [18] numerically investigated the impacts of torque and angular speed to transverse asymmetric vibration of the connection disk. He et al. [19] analyzed the vibration of a rotating elastic connection disk under the effects of small axial displacement and deflection angles due to machining and installation errors. Moreover, in Bauer and Eidel [20], the transverse vibration and stability of rotating solid circular plate was investigated under 4 outer edge boundary conditions, but the solid

* Corresponding author.
E-mail address: peiyc@jlu.edu.cn (Y.-C. Pei).

https://doi.org/10.1016/j.ijmecsci.2018.01.013
Received 19 December 2016; Received in revised form 12 April 2017; Accepted 10 January 2018
Available online 31 January 2018
0020-7403/

Nomenclature

a	disk outer radius
$a_{m,n}$	initial runout amplitude
$A_{m,n}$	steady state amplitude
b	disk inner radius
diag	diagonal matrix operator
E	Young's modulus
$g_{m,n}$	steady state amplitude ratio
h	disk plate thickness
Im	complex imaginary part operator
I_n	modified Bessel function
J_n	Bessel function
K_n	modified Bessel function
m	mode nodal circle number
M_r	radial transverse moment
n	mode nodal diameter number
N_r	radial stress resultant
$N_{r\theta}$	shear stress resultant
N_θ	circumferential stress resultant
p_{Cr}	dimensionless lowest critical speed
p_{Lm}	dimensionless lowest limit speed
r	radial coordinate
R	radial edge coordinate
Re	complex real part operator
S_{Cr}	lowest critical speed
S_{Lm}	lowest limit speed
S_r	radial edge force
T	transmitted torque
T_R	transmitted torque reference
u_r	radial displacement
u_θ	circumferential displacement
w	total transverse deflection
$\tilde{w}$	initial transverse runout
W_n	radial mode shape
Y_n	Bessel function
ζ	dimensionless rotating speed
η	disk inner and outer radius ratio
θ	circumferential coordinate
ν	Poisson's ratio
ξ	dimensionless natural frequency
ρ	mass density
τ	dimensionless transmitted torque
ω_n	fundamental natural frequency
ω_R	fundamental frequency reference
Ω	rotating angular speed
Ω^n_{Cr}	critical speed of nodal diameter n
Ω^n_{Lm}	limit speed of nodal diameter n
Type C	stiff clamped (fixed) boundary
Type F	completely free boundary
Type P	slip clamped boundary
Type R	rolling supported boundary
Type S	simply supported (rolling clamped) boundary
Type X	someone boundary of type C, F, P, R and S
Constant	normal upright 'x'
Matrix	bold upright '**x**'
Variable	normal italic '*x*'
Vector	bold italic '***x***'

circular plate cannot have the inner edge and corresponding boundary conditions as the rotating annular disk.

In the view of practical mechanical structures, there are several potential inner and outer edge boundary conditions that can be applied in the rotating disk: stiff clamped (fixed), simply supported (rolling clamped), slip clamped, rolling supported, and completely free from the plate and shell theory [21–23]. However, the rotating disk dynamics characteristics corresponding to these potential boundary conditions have not been recognized and mastered systematically until now. It is a problem that how to select a proper edge boundary of rotating disk to make the disk work stably and efficiently.

In this paper, 25 full combinations of edge boundary conditions are recognized and compared with each others, their natural frequency, dynamic stability, critical and limit speeds, and steady state response amplitude under initial transversal runout are investigated. This paper is very meaningful and beneficial to select the most available and suitable scheme of edge boundary conditions in the rotating disk application.

2. Fundamental governing model

An annular flexible disk with inner and outer radius b and a rotates at a constant angular speed Ω, and the disk thickness h is very small compared with its inner and outer radius. A small vibration of disk transverse deflection is considered in this paper.

In a polar coordinate system (r, θ), the force-displacement relationships [21–23] are,

$$N_r = \frac{Eh}{1-\upsilon^2}\left[\frac{\partial u_r}{\partial r} + \upsilon\left(\frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta}\right)\right] \tag{1}$$

$$N_\theta = \frac{Eh}{1-\upsilon^2}\left[\left(\frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta}\right) + \upsilon\frac{\partial u_r}{\partial r}\right] \tag{2}$$

$$N_{r\theta} = \frac{Eh}{2(1+\nu)}\left(\frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\right) \tag{3}$$

$$M_r = -\frac{Eh^3}{12(1-\nu^2)}\left[\frac{\partial^2 w}{\partial r^2} + \nu\left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}\right)\right] \tag{4}$$

$$S_r = -\frac{Eh^3}{12(1-\nu^2)}\left[\frac{\partial}{\partial r}\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}\right) + \frac{1-\nu}{r^2}\frac{\partial^2}{\partial \theta^2}\left(\frac{\partial w}{\partial r} - \frac{w}{r}\right)\right] \tag{5}$$

where E, ν and ρ are the Young's modulus, Poisson's ratio and mass density of the disk; u_r and u_θ are the radial and circumferential in-plane displacements of the middle plane; N_r, N_θ and $N_{r\theta}$ are the in-plane stress resultants; M_r and S_r are the radial transverse moment and edge force [22].

And the equations of equilibrium [19,22] are

$$\frac{1}{r}\frac{\partial N_\theta}{\partial \theta} + \frac{\partial N_{r\theta}}{\partial r} + 2\frac{N_{r\theta}}{r} = 0 \tag{6}$$

$$\frac{\partial N_r}{\partial r} + \frac{1}{r}\frac{\partial N_{r\theta}}{\partial \theta} + \frac{N_r - N_\theta}{r} + \rho h\Omega^2 r = 0 \tag{7}$$

In the polar coordinate system (r, θ) fixed on the ground, the small transverse vibration governing equation of the flexible disk with an initial unstressed transverse runout $\tilde{w}$ [19,24,25] can be established as

$$\rho h\left(\frac{\partial^2 w}{\partial t^2} + 2\Omega\frac{\partial^2 w}{\partial t\partial \theta} + \Omega^2\frac{\partial^2 w}{\partial \theta^2}\right) + \frac{Eh^3}{12(1-\nu^2)}\nabla^4(w-\tilde{w}) = \frac{\partial}{r\partial r}\left(rN_r\frac{\partial w}{\partial r}\right) + \frac{\partial}{r^2\partial \theta}\left(N_\theta\frac{\partial w}{\partial \theta}\right) + \frac{\partial}{r\partial r}\left(N_{r\theta}\frac{\partial w}{\partial \theta}\right) + \frac{\partial}{r\partial \theta}\left(N_{r\theta}\frac{\partial w}{\partial r}\right) \tag{8}$$

where w is the disk total deflection; $\nabla^4 = (\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2})^2$ is a biharmonic differential operator.

3. Edge boundary assignment

In mechanical engineering, there are five usual edge boundary conditions [21,23] in Fig. 1 at $r = R$ with $R = a$ or b for the rotating flexible annular disk,