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Partial slip problem in frictional contact of orthotropic elastic half-plane and rigid punch

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ABSTRACT

Planar partial-slip contact problem of a rigid indenter with flat end and an orthotropic elastic material is analytically and numerically investigated. In the analytical way, the coupled singular equations of this problem are reduced to a Fredholm integral equation with a regular kernel. The analytical solutions are derived in the forms of the Goodman's and Spence's approximations. In the numerical way, a linear complementarity formulation is developed by reformulating the governing equations are as coupled Volterra integral equations. And, a Newton-based optimization algorithm based on smoothing approximation is used to solve the problem. The validness of both the Goodman and Spence approximate solutions is verified by comparing with the numerical results for different orthotropic elastic materials.

1. Introduction

Contact between a rigid punch and orthotropic materials can be found in various engineering applications such as instrumented indentation test [1], tactile sensor design [2], metal forming [3]. The knowledge of deformation states during contact is essentially important for better understanding these application processes. Although prior investigations to contact problem have generated amounts of valuable results since the seminal Hertz's work about one and a half century ago, most of the results are restricted to isotropic elastic solids [4]. And, contact problem of anisotropic materials is still far way from completeness. It is well known that the general solution in anisotropic elasticity can be represented in the Lekhnistskii formulism or the Stroh formulism. As a direct generalization of Muskhelishvili's approach for planar isotropic elasticity , the Lekhnistskii formulism is developed in terms of elastic compliances. On the other hand, Stroh formulism placing its basis on elastic stiffnesses is easy to generalize to three-dimensional analysis [5]. Based on the two formulisms, many researchers studied contact processes of anisotropic materials with different boundary conditions.

By using the Lekhnistskii formulism, [6] simplified the frictionless and the frictional sliding contact problems to be two particular cases of the Hilbert problem and presented corresponding analytical solutions. Based on stress functions corresponding to normal and tangential tractions, [7] presented an iterative scheme for a flat-end punch sliding over an anisotropic half plane, in which the mutual influences of normal and tangential tractions on each other are neglected in turn. Based on singular equations governing surface tractions and surface displacement gradient, [8] obtained closed-form solutions for indentation of an anisotropic elastic half plane by a flat-end punch under four different boundary conditions. Utilizing the perturbation technique, [9] presented an asymptotic solution for indentation of a parabolic indenter against an orthotropic elastic layer. By using the Stroh formulism, [10] investigated bond contact between a rigid punch with arbitrary profile and an anisotropic elastic half-plane. By splitting the entire contact domain between a rigid indenter and an elastic layer into several regions with Dirichelet or Neumann boundary conditions, [11] presented a technique to find an analytical solution in terms of a series form. Based on the Fourier integral transform, [12] analytically solved the sliding frictional contact of a flat-end or parabolic punch and an orthotropic half plane. Furthermore, there are a lot of research literatures concerning on contact of anisotropic piezoelectric materials [13] and graded solids [14]. However, in the above mentioned literatures, the studied contact problems are either frictionless or sliding frictional.

To treat the partial slip contact arisen in normal indentation of an isotropic half-space in presence of finite friction, it needs to treat the boundary conditions imposed on the stick and slip regions. Since the stick/slip boundary appears as an additional unknown parameter, hence the solution procedure becomes much more complex than the frictionless problem. In practical applications, the Goodman's approximation, in which the influence of the shear stress on the normal stress is neglected, is widely used [15]. For example, based on the Goodman's approxima-

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tion, [16] considered the frictional indentation of a functionally graded coated half-space by a rigid punch, [17] studied the frictional contact between two elastic cylinders. Based on the self-similarity assumption, [18] found that the slip radius is the same for all power-law indenters. Then, [18] compared the results evaluated by the Goodman's approximation and the coupled integral equations governing the partial slip contact. By using planar bipolar coordinates, [19,20] reduced the problem as a singular equation in terms of the normal stress in the slip region, and presented an analytical solution for the indentation of a rigid cylinder or sphere on an elastic half space. In addition, a few papers paid attention to the partial slip contact in the Cattaneo problem [21,22].

In this paper, the partial contact problem between a rigid punch and an orthotropic elastic half plane is studied. Our primary aims are to investigate the influences of the friction force and the material orthotropy on the splitting boundary between the stick and slip regions, and whether the Goodman's and Spence's approximations are valid for the orthotropic elastic solids. The rest of the present paper is organized as follows. In Section 2, the formulation of the problem and the surface Green's function of the orthotropic elastic half plane are briefly described. Section 3 gives the analytical Goodman's and Spence's approximations based on the integral equation governing the partial slip contact of a flat-end punch and a orthotropic elastic half plane. Then, an efficient numerical scheme for determining the contact stresses and the slip/stick boundary is presented in Section 4. Numerical results are provided in section 5 to show the influence of the material anisotropy. Finally, conclusions are drawn in Section 6.

2. Problem statement

2.1. Surface Green's function

Considering an orthotropic half plane with its principle axes of orthotropy aligning with the Cartesian coordinates (x, y) , the two dimensional strain-stress relation can be expressed as

$$\begin{bmatrix}\epsilon_{xx}\\ \epsilon_{yy}\\ \epsilon_{xy}\end{bmatrix}=\begin{bmatrix}s_{11} & s_{12} & 0\\ s_{12} & s_{22} & 0\\ 0 & 0 & s_{66}\end{bmatrix}\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{bmatrix}=\frac{1}{E}\begin{bmatrix}\delta^{-2} & -\nu & 0\\ -\nu & \delta^2 & 0\\ 0 & 0 & \kappa+\nu\end{bmatrix}\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{bmatrix} \tag{1}$$

where E, ν, δ and κ are the effective stiffness, the effective Poisson's ratio, the stiffness ratio and the shear parameter, respectively. These four parameters have the following relationships with the four engineering constants E_{11}, E_{22}, G_{12} and ν_{12}

$$E=\sqrt{E_{11}+E_{22}},\qquad \nu=\sqrt{\nu_{12}\nu_{21}},\qquad \delta^4=\frac{E_{11}}{E_{22}},\qquad \kappa=\frac{E}{2G_{12}}-\nu \tag{2}$$

for the plane-stress case, and

$$E=\sqrt{\frac{E_{11}+E_{22}}{(1-\nu_{13}\nu_{31})(1-\nu_{23}\nu_{32})}},\quad \nu=\sqrt{\frac{(\nu_{12}+\nu_{13}\nu_{32})(\nu_{21}+\nu_{23}\nu_{31})}{(1-\nu_{13}\nu_{31})(1-\nu_{23}\nu_{32})}},$$
$$\delta^4=\frac{E_{11}}{E_{22}}\frac{1-\nu_{23}\nu_{32}}{1-\nu_{13}\nu_{31}},\quad \kappa=\frac{E}{2G_{12}}-\nu \tag{3}$$

for the plane-strain case.

It is well known that the Airy stress function $\Phi(x, y)$, which automatically satisfies the equilibrium equations, is particularly suitable for the two-dimensional elasticity. And, Appendix A detailedly presents the elastic responses corresponding to concentrated force. Based on Eqs. (A.19) and (A.20), for the orthotropically elastic half plane subjected to arbitrary distributed loadings $f_x(x)$ and $f_y(x)$, the surface displacement gradients have the following expressions

$$-u_0'(x)=Af_y(x)+B\int_b^a\frac{f_x}{x-s}\mathrm{d}s \tag{4}$$

$$-v_0'(x)=C\int_b^a\frac{f_y}{x-s}\mathrm{d}s-Af_x(x) \tag{5}$$

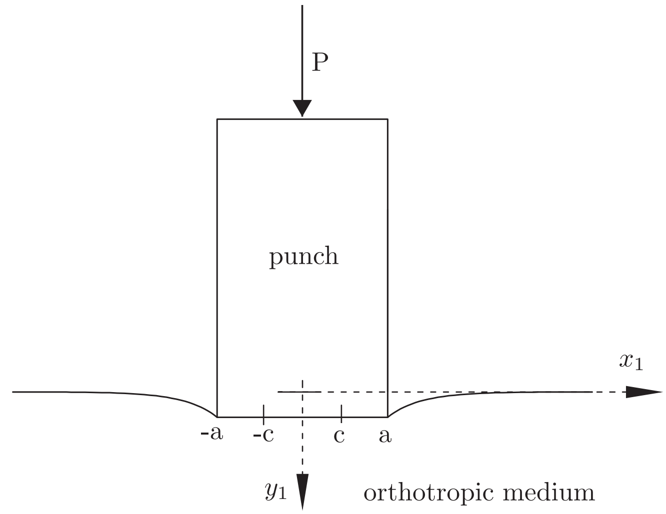

Fig. 1. Partial slip indentation of flat-ended punch on orthotropic medium.

2.2. Indentation of flat punch

In a Cartesian coordinates (x_1, y_1), let a rigid punch with a flat-end profile be brought to contact with an elastic half plane $y_1<0$ by normal force P over the contact area $-a\le x_1\le a$, as shown in Fig. 1. After the variable changes $x=ax_1$, $y=y_1$, the yielded normal and shear stresses at the contact area can be expressed as

$$(\sigma_{yy})_{y=0}=-\frac{P}{a}p(x),\qquad (\sigma_{xy})_{y=0}=\frac{P}{a}q(x) \tag{6}$$

where $p(x)$ and $q(x)$ are the normal and shear surface tractions for the indentation caused by a punch with flat end $-1<x<1$ subjected to the unit normal force, respectively. Due to the effect of the friction force, the total contact area is split to the slip region and the stick region. As a result, the contact boundary conditions are defined as

$$v_0'(x)=0,\qquad |x|\le 1 \tag{7}$$

$$u_0'(x)=0,\qquad |x|<c \tag{8}$$

$$p(x)-q(x)/\mu=0,\qquad c\le|x|\le 1 \tag{9}$$

where μ is the friction coefficient, and c represents the extent of the slip region i.e., $|x|<c$ and $c\le|x|\le1$ are the stick and slip regions, respectively. Following the relations in Eqs. (4) and (5), the boundary conditions in Eqs. (7)–(9) can be expressed in terms of $p(x)$ and $q(x)$ as the following Fredholm integral equations

$$C\int_{-1}^{1}\frac{p(t)}{x-t}dt-Aq(x)=0,\qquad |x|\le 1 \tag{10}$$

$$B\int_{-1}^{1}\frac{q(t)}{x-t}dt+Ap(x)=0,\qquad |x|<c \tag{11}$$

where u' is positive. Considering Eqs. (1), (10) and (11), we can see that the effective stiffness E has no influence on the stress distribution in the slip contact area. Taking account of the governing equations for an isotopic half plane

$$\frac{1}{\pi}\int_{-1}^{1}\frac{p(t)}{t-x}\mathrm{d}\,t+\wp q(x)=0,\qquad |x|\le 1 \tag{12}$$

$$\frac{1}{\pi}\int_{-1}^{1}\frac{q(t)}{t-x}\mathrm{d}\,t-\wp p(x)=0,\qquad |x|<c \tag{13}$$

where $\wp=(1-2\nu)/(2-2\nu)$ with ν as the Poisson's ratio. It is easy to find that when $B=C$, the orthotropic elasticity reduces to be the isotropic elasticity. And, B/A, C/A are two characteristic parameters for the frictional contact.