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Size-dependent pull-in analysis of a composite laminated micro-beam actuated by electrostatic and piezoelectric forces: Generalized differential quadrature method

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ABSTRACT

In this paper, a size-dependent model of clamped-clamped composite laminated electrostatic Euler–Bernoulli microbeams with piezoelectric layers attached was proposed based on a new modified couple stress theory for anisotropic elasticity. With the aid of Hamilton's principle, the nonlinear differential governing equation was established and solved by utilizing the generalized differential quadrature method. The present model, which can be reduced to the isotropic beam model takes the residual stresses, fringing effects and midplane stretching effects into consideration. A good agreement exists between numerical results of the present reduced model and those of available literature. Numerical analysis reveals that the size effect is significant when geometry sizes of a beam are comparable to the internal material length scale parameter (hereafter referred to as MLSP). Furthermore, the pull-in voltage dependence of the residual stress, midplane stretching, MLSP as well as the voltage applied to the piezoelectric layers are also investigated. The present model can accurately predict the static behavior and guide the design of composite laminated micro-beam actuated by both the electrostatic and piezoelectric forces.

1. Introduction

In the past decades, micro-electro-mechanical system (MEMS) devices are widely used in various situations such as aerospace, information technology, high and new technology, as well as biomedical industries. MEMS devices are generally classified by their actuation mechanisms. And electrostatic, thermal, pneumatic and piezoelectric actuated MEMS devices are widely used [1]. Electrostatically actuated MEMS devices demand few mechanical parts and lower power consumption and are more widely applied in MEMS. Thereinto, microbeams are typical building blocks in electrostatically actuated MEMS devices.

A typical electrostatically actuated device consists of two conductive electrodes in which the movable electrode is suspended above the fixed one (Fig. 1) with an applied direct current voltage. Owe to the electrostatic attraction, the former deforms toward the latter. When a direct current (DC) is applied between the microbeam and electrode, the movable microbeam deflects towards to the fixed electrode owing to the electrostatic actuation. The electrostatic attraction increases with the applied DC voltage in a nonlinear fashion, and the critical voltage gives rise to the failure of the movable electrode i.e. pull-in instability. The thresholds of the voltage and displacement corresponding to the unstable collapsing are respectively called the pull-in voltage and the pull-in displacement [2]. Hence, correct predications of the pull-in voltage and the pull-in displacement are the main problems in the design of electrostatically actuated MEMS devices.

In the past decades, the characterization of electrostatically actuated microbeams has received considerable interests. The pull-in characterization of cantilever, fixed-fixed and circulate plate microstructures were investigated by Osterberg et al. [3]. Abel-Rahman et al. [4] presented a dynamical model including the midplane stretching and the nonlinear electric force of microbeams. Younis et al. [5] developed a reduced order model to investigate the size-dependent characteristics of the electrostatically actuated microbeam. Sadeghian et al. [6] took the influence of residual stress, midplane stretching and fringing effects into consideration and used the generalized differential quadrature method (GDQM) to solve the governing differential equations. Hsu [7] investigated the pull-in behavior of the cantilever actuators accounting for the electrode shapes, the lengths and the moments of inertia of the actuators applying the GDQM. However, the abovementioned investigations are conducted by using the classical continuum theory.

To our knowledge, the geometry sizes of MEMS devices typically range from microns or even sub-microns. In recent years, some experiments were conducted and the size-dependent mechanical behaviors

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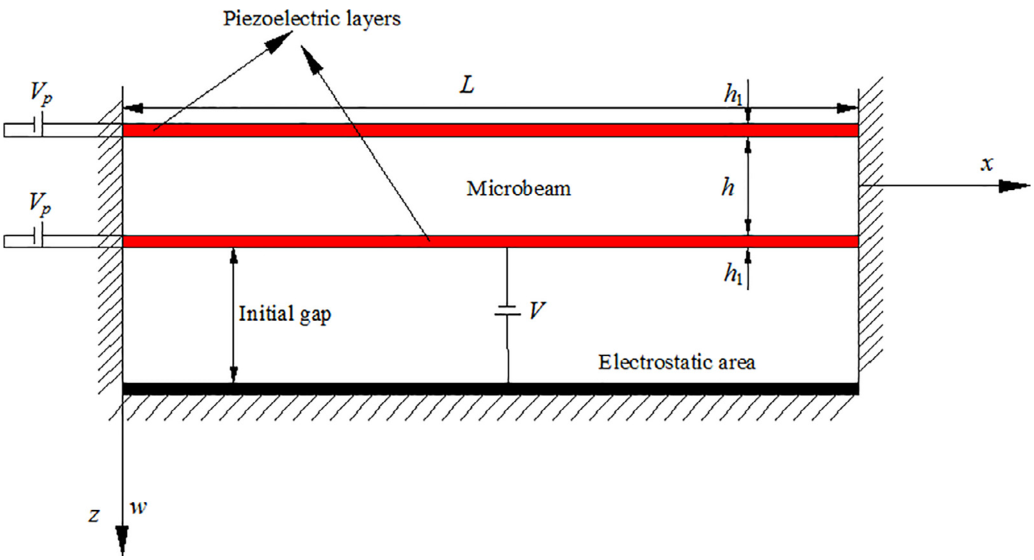

Fig. 1. Schematic of fixed-fixed MEMS actuator with piezoelectric layers.

were observed in different micro-structures [8–12]. The results of these experiments reveal that classical beam models meet significant difficulties in explaining the size-dependent behaviors because of the absence of material length scale parameters. Hence, non-classical continuum elasticity theories should be used to capture size effects such as the nonlocal theory [13], the strain gradient theory [14], the couple stress theory [15], the modified couple stress theory [26], the modified strain gradient theory [10] and the consistent couple stress theory [16]. Various studies have been conducted to investigate the size-dependent mechanical behaviors of small-scale structures applying these non-classical elasticity theories [17–28]. Due to having only one internal MLSP and the symmetry of the couple stress tensor, the modified couple stress theory established by Yang et al. [29] has activated a surge of scientific interests in recent years. Plenty of models have been established based on the modified couple stress theory to analyze size-dependent mechanical characteristics of micro-structures under different beam theories. For instance, a size-dependent Euler–Bernoulli beam was developed for the static bending by Park and Gao [30] and for the dynamic problems by Kong et al. [31] on the basis of the modified couple stress theory, respectively. Ma et al. [32] proposed a microstructure-dependent Timoshenko beam model based on the modified couple stress theory for the static bending and free vibration problems. Arbind et al. [33] developed a microstructure-dependent nonlinear third-order theory for analysis of functionally graded beams based on the modified couple stress theory. Also, Mohammadabadi et al. [34] studied the thermal effect on size-dependent buckling analysis of micro composite laminated beams, and the Euler–Bernoulli, Timoshenko, and Reddy beam theories were considered.

Besides, the modified couple stress theory was also used to predict the size-dependent pull-in behaviors of electrostatically actuated microstructures [35–39]. Yin et al. [35] used the generalized differential quadrature method to explore the size effect of electrostatically actuated microbeams. Abdi et al. [36] developed an electrostatic cantilever microbeam model to research the static pull-in behavior. Beni et al. [37] conducted a theoretical investigation on the effects of the pull-in instability of the beam-type NEMS. Shaat et al. [38] proposed a couple stress based-Euler–Bernoulli beam model for pull-in and bio-mass sensing applications. And Kazemi et al. [39] investigated the nonlinear pull-in instability of microplates with piezoelectric layers. The available investigations demonstrate that the electrostatically actuated MEMS devices have several significant deficiencies such as high driving voltage and low reliability [40,41].

Recently, the applications of smart materials in MEMS have drawn serious attention from the scientific community. The piezoelectric materials possess numerous profits containing light weight, high operating bandwidth, quick-response, low power consumption and abilities of generating very small motions and high force transmission as well [2,39]. Therefore, piezoelectric materials have been implemented to reach higher efficiency or to decrease the pull-in voltage in electrostatically actuated MEMS devices [2,39–42]. For instance, Xiao et al. [2] and Raeisifard et al. [41] intended to adjust the pull-in voltage of electrostatically actuated microbeams by attaching piezoelectric layers. They respectively presented the size-dependent models for fixed-fixed and cantilever microbeam, and used the Galerkin method to solve the governing differential equations.

On the other hand, composite laminated materials are increasingly used in engineering structures due to the larger ratios of strength-to-weight and stiffness-to-weight [43–48]. However, all of the aforementioned models [2–7,35–42] are limited to isotropic theory and are unable to solve anisotropic problems, especially establishing couple stress based laminated beam/plate models. Indeed, it is not straightforward to extend the couple stress theory to establish composite laminated models. Recently, Chen et al. [49,50] defined a new curvature tensor to represent the constitutive equations of laminated beam/plate for anisotropy materials, which can be degenerated to the isotropic modified couple stress based model. In the present study, to the authors' best knowledge, a size-dependent model to analysis size-dependent pull-in performance of composite laminated microbeams actuated by electrostatic and piezoelectric forces is established for the first time. The main purpose of the present paper is to derive a new size-dependent electrostatically actuated clamped-clamped composite laminated Euler–Bernoulli beam model with piezoelectric layers based on the modified couple stress theory for anisotropy materials. The outline of the present paper is organized as follows. In Section 2, the nonlinear differential governing equation is presented with the aid of Hamilton's principle. To study the pull-in voltage of composite laminated electrostatic MEMS actuators, numerical analysis and discussions applying the GDQM and the Newton–Raphson method are proposed in Section 3. Section 4 summarized the major conclusions.

2. Formulation

2.1. The modified couple stress theory for isotropic materials

Based on the modified couple stress theory [29], the symmetrical strain tensor and curvature tensor can be expressed as follow:

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \tag{1}$$