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Investigation of non-local theory solution to a three-dimensional rectangular permeable crack in magneto-electro-elastic materials

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ABSTRACT

This paper presents the non-local theory solution to a three-dimensional rectangular permeable crack in magneto-electro-elastic materials (MEEMs) using the generalized Almansi's theorem and the Schmidt method. The problems are formulated through Fourier transform as three pairs of dual integral equations, in which the unknown variables are the jumps of elastic displacement, electric potential and magnetic potential jumps across the crack surfaces. The displacement jumps across the crack surfaces are directly expanded as a series of Jacobi polynomials to solve the dual integral equations and the resulting equations are solved using the Schmidt method. Numerical examples are provided to show the effects of the geometric shape of rectangular crack and the lattice parameter on the stress, the electric displacement and the magnetic flux fields near the crack edges in magneto-electro-elastic materials. Unlike the classical solution, the present solutions exhibit no stress, electric displacement and magnetic flux singularities near the crack edges in magneto-electro-elastic materials.

1. Introduction

The excellent magnetic-electric-mechanical coupling effects of the magneto-electro-elastic materials (MEEMs) have been extensively used in a variety of engineering structures, such as supersonic aircraft, actuators, sensors, nuclear reactors, nuclear submarines, electronic packaging, etc [1,2]. Because of the brittleness of MEEMs, fracture analyses of MEEMs are very important for structural design and application. Consequently, some main achievements on the static and dynamic fracture problem have been studied [2-9]. Liu et al. [1] investigated the fracture behaviors of magnetoelectroelastic cylinder induced by a penny-shaped magnetically dielectric crack. Li et al. [2] studied an elliptical planar crack embedded in an infinite transversely isotropic medium in the framework of magneto-electro-elasticity. Liu et al. [8] analyzed four three-dimensional rectangular cracks in magneto-electro-elastic material by using the generalized Almansi's theorem and the Schmidt method under limited-permeable boundary conditions. The problem of a penny-shaped crack subjected to symmetric uniform heat flux in an infinite transversely isotropic magneto-electro-thermo-elastic medium was researched by Yang et al [10]. All the above works have a common point that there exists stress singularity at the crack tips or along the rectangular crack edges based on the classical edacity theory. According to the physical nature in practical engineering, the stress fields at the crack tips or the rectangular crack edges should be finite. Thus, beginning with Eringen [11], the non-local theory was used to investigate the fracture problem in the elastic materials.

The modern non-local continuum mechanics have been proposed in the past [11–13]. To overcome the stress singularity at the crack tips, Eringen et al. [14] investigated the stress near the tip of a sharp line crack in an isotropic elastic plate subject to uniform tension by using non-local theory. In the previous works [15–19], the arising of singular stresses and strains at the tip of a crack by using the conventional theory of elasticity has been effectively overcome in the following ways: a) By disregarding of the stresses and strains at the crack tip and focusing the attention on a stress field parameter, mathematically defined in order to "cancel" the singular nature of stresses [15]; b) By using averaged parameters, which are always finite [16–18]; c) and using new formulations to describe the behavior of elastic solids and within the context of a strain limiting theory of elasticity [19].

In recently, Zhou et al. [20] analyzed two collinear mode-I permeable cracks in a magnetoelectroelastic composite material plane by using non-local theory and the Schmidt method [21,22]. Jamia et al. [23] studied the problem of a mixed-mode crack embedded in an infinite medium made of a functionally graded magneto-electro-elastic material (FGMEEM) with the crack surfaces subjected to magneto-

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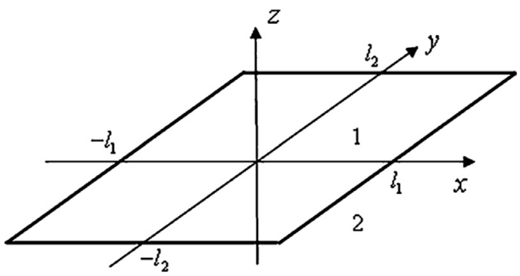

Fig. 1. Geometry and coordinate system for a rectangular crack.

electro-mechanical loadings by use of the non-local theory. Ma et al. [24] made the first attempt to investigate the dispersion behavior of waves in magneto-electro-elastic (MEE) nanobeams. Liu et al. [25] addressed the non-local theory solution for a 3D rectangular permeable crack in piezoelectric composite materials under a normal stress loading by the Schmidt method. According to the literature survey, the magneto-electro-elastic behavior of a three-dimensional (3D) rectangular permeable crack in MEEMs under a uniform tension loading has not been considered by means of non-local theory.

In the present paper, the non-local theory solution of a 3D rectangular permeable crack in MEEMs subjected to a normal stress loading is studied by using the generalized Almansi's theorem and the Schmidt method. The structure of this paper is as follows. In Section 2, the basic equations of the non-local MEEMs are presented and the problem with the corresponding boundary conditions is described. Three pairs of dual integral equations are established in Section 3. In Section 4, the non-local stress, non-local electric displacement and non-local magnetic flux fields are obtained. The numerical examples and discussions are given in Section 5. Finally, the conclusions are drawn in Section 6.

2. Problem description and formulation

Assume that there is a symmetric rectangular cracks located at $z=0$ along the x-axis from $-l_1$ to l_1 and along the y-axis from $-l_2$ to l_2 in a transversely isotropic MEEMs (Fig. 1). A Cartesian coordinate system (x, y, z) is positioned, with the plane $x-y$ parallel to the plane of isotropy. Considered a distributed normal stress loading $\sigma^*_{zz}(x,y,0)=-\sigma_0$ (here σ_0 is the magnitude of the uniform tension stress loading) is directly applied on the crack surfaces, which is equivalent to investigate the perturbation fields for a remotely loaded cracked-body through the standard superposition technique in fracture mechanics.

2.1. Basic equations of the non-local magneto-electro-elastic materials

For 3D non-local transversely isotropic MEEMs, the basic equations of linear, homogeneous, the absence of body forces, electric and magnetic charges are stated as

$$\sigma^*_{ik,k}=0,\quad D^*_{i,i}=0,\quad B^*_{i,i}=0,\ (i,k=x,y,z) \tag{1}$$

$$\sigma^*_{ik}(x,y,z)=\int_V \sigma_{ik}(x',y',z')\alpha(|x'-x|,|y'-y|,|z'-z|)dV(x',y',z'),\quad (i,k=x,y,z) \tag{2}$$

$$D^*_{i}(x,y,z)=\int_V D_{i}(x',y',z')\alpha(|x'-x|,|y'-y|,|z'-z|)dV(x',y',z'),\quad (i,k=x,y,z) \tag{3}$$

$$B^*_{i}(x,y,z)=\int_V B_{i}(x',y',z')\alpha(|x'-x|,|y'-y|,|z'-z|)dV(x',y',z'),\quad (i,k=x,y,z) \tag{4}$$

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{zz}\\ \sigma_{xy}\\ \sigma_{xz}\\ \sigma_{yz}\end{Bmatrix}=\begin{bmatrix} c_{11}\partial_{,x} & c_{12}\partial_{,y} & c_{13}\partial_{,z} & e_{31}\partial_{,z} & q_{31}\partial_{,z}\\ c_{12}\partial_{,x} & c_{11}\partial_{,y} & c_{13}\partial_{,z} & e_{31}\partial_{,z} & q_{31}\partial_{,z}\\ c_{13}\partial_{,x} & c_{13}\partial_{,y} & c_{33}\partial_{,z} & e_{33}\partial_{,z} & q_{33}\partial_{,z}\\ (c_{11}-c_{12})\partial_{,y}/2 & (c_{11}-c_{12})\partial_{,x}/2 & 0 & 0 & 0\\ c_{44}\partial_{,z} & 0 & c_{44}\partial_{,x} & e_{15}\partial_{,x} & q_{15}\partial_{,x}\\ 0 & c_{44}\partial_{,z} & c_{44}\partial_{,y} & e_{15}\partial_{,y} & q_{15}\partial_{,y}\end{bmatrix}\begin{Bmatrix}u\\ v\\ w\\ \phi\\ \psi\end{Bmatrix} \tag{5}$$

$$\begin{Bmatrix}D_x\\ D_y\\ D_z\end{Bmatrix}=\begin{bmatrix} e_{15}\partial_{,z} & 0 & e_{15}\partial_{,x} & -\varepsilon_{11}\partial_{,x} & -d_{11}\partial_{,x}\\ 0 & e_{15}\partial_{,z} & e_{15}\partial_{,y} & -\varepsilon_{11}\partial_{,y} & -d_{11}\partial_{,y}\\ e_{31}\partial_{,x} & e_{31}\partial_{,y} & e_{33}\partial_{,z} & -\varepsilon_{33}\partial_{,z} & -d_{33}\partial_{,z}\end{bmatrix}\times\{u\quad v\quad w\quad \phi\quad \psi\}^T \tag{6}$$

$$\begin{Bmatrix}B_x\\ B_y\\ B_z\end{Bmatrix}=\begin{bmatrix} q_{15}\partial_{,z} & 0 & q_{15}\partial_{,x} & -d_{11}\partial_{,x} & -\mu_{11}\partial_{,x}\\ 0 & q_{15}\partial_{,z} & q_{15}\partial_{,y} & -d_{11}\partial_{,y} & -\mu_{11}\partial_{,y}\\ q_{31}\partial_{,x} & q_{31}\partial_{,y} & q_{33}\partial_{,z} & -d_{33}\partial_{,z} & -\mu_{33}\partial_{,z}\end{bmatrix}\times\{u\quad v\quad w\quad \phi\quad \psi\}^T \tag{7}$$

where σ^*_{ik}, D^*_i and B^*_i are the non-local stresses, non-local electric displacements and non-local magnetic flux, while σ_{ik}, D_i and B_i represent the stress tensor, electric displacement tensor and magnetic flux tensor for a classical (i.e. local) constitutive equations; $u(x, y, z)$, $v(x, y, z)$ and $w(x, y, z)$ represent the displacement components in the $x-$, $y-$and $z-$axis directions, $\phi(x, y, z)$ is the electric potential and $\psi(x, y, z)$ is the magnetic potential; c_{11}, c_{12}, c_{13}, c_{33} and c_{44} are classical elastic stiffness constants, ε_{11} and ε_{33} are dielectric constants, e_{15}, e_{31} and e_{33} are piezoelectric constants, d_{11} and d_{33} are magnetoelectric constants, q_{15}, q_{31} and q_{33} are piezomagnetic constants and μ_{11} and μ_{33} are magnetic permeability constants in the transversely isotropic MEEMs. The only difference from the classical MEEMs lie in the stress, electric displacement and magnetic flux constitutive Eqs. (2)–(7), in which the non-local stresses σ^*_{ik}, the non-local electric displacement D^*_i and non-local magnetic flux B^*_i at a point (x, y, z) depends on the strains $u_{,k}$, $v_{,k}$, $w_{,k}$, $\phi_{,k}$ and $\psi_{,k}$ ($k=x, y, z$) at all points (x', y', z') in the body having volume V, bounded by ∂V. Here, $\alpha(|x'-x|, |y'-y|, |z'-z|)$ is a nonlocal kernel called the influential function. As discussed in Eringen and Kim [26], the expression is given as

$$\alpha(|x'-x|,|y'-y|,|z'-z|)=\alpha_0\exp\{-(\beta/a)^2[(x'-x)^2+(y'-y)^2+(z'-z)^2]\} \tag{8}$$

where β and a are material constants. Normally, a is an internal characteristic length (e.g., lattice parameter, granular distance) and β is an external characteristic length (e.g., crack length, wavelength) [27]. In the present paper, a is taken as the lattice parameter, and α_0 is determined by the normalization

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\alpha(|x'-x|,|y'-y|,|z'-z|)dx'dy'dz'=1 \tag{9}$$

Substituting Eq. (8) into Eq. (9), α_0 is obtained as

$$\alpha_0=\pi^{-3/2}(\beta/a)^3 \tag{10}$$

Substituting Eqs. (2)–(10) into Eq. (1) and using the Green–Gauss theorem, we have

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\alpha(|x'-x|,|y'-y|,|z'-z|)\times\left(\frac{\partial\sigma^{(j)}_{xx}(x',y',z')}{\partial x'}+\frac{\partial\sigma^{(j)}_{xy}(x',y',z')}{\partial y'}+\frac{\partial\sigma^{(j)}_{xz}(x',y',z')}{\partial z'}\right)dx'dy'dz' - \int_{-l_1}^{l_1}\int_{-l_2}^{l_2}\alpha(|x'-x|,|y'-y|,0)[\sigma^{(1)}_{xz}(x',y',0)-\sigma^{(2)}_{xz}(x',y',0)]dx'dy'=0 \tag{11}$$