International Journal of Mechanical Sciences 117 (2016) 334–345

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Non-parametric free-form optimal design of frame structures in natural frequency problem

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ARTICLE INFO

Article history:
Received 27 September 2015
Received in revised form
1 August 2016
Accepted 31 August 2016
Available online 2 September 2016

Keywords:
Frame structure
Free-form
Non-parametric
Optimal design
Repeated eigenvalue
Vibration

ABSTRACT

Optimal design of structures with respect to their mechanical behavior is essential and basically required in structural engineering. In this study, we propose a non-parametric free-form optimization method based on the variational method to design frame structures composed of arbitrarily curved linear elastic members. The natural frequency maximization problem of frame structures is formulated as a non-parametric shape optimization problem under the volume constraint. Under the assumption that each member varies in the out-of-plane direction to its centroidal axis, the shape gradient functions and the optimality conditions are theoretically derived by the Lagrange multiplier method and the formulae of the material derivative. Then, the derived shape gradient functions are applied to a gradient method in the Hilbert space with a P.D.E (Partial Differential Equation) smoother, which is referred as the H^1 gradient method for frame structures. Moreover, a simple switching technique of the objective functional is presented for overcoming the discontinuity problem of repeated eigenvalues, which often appears in natural frequency maximization problem. With this combination of the three techniques, the optimal free-form frame structures owning smoothly curved members can be obtained without any preliminary shape parameterization, and the effectiveness and validity of the proposed method are verified through three design examples.

1. Introduction

Up to now, mechanical properties of structures, such as stiffness, vibration and buckling analyses, have been studied for hundreds of years, and are still under interest among scholars by developing new mechanical methods or designing optimal structures. Structures composed of straight or curved beams are classified as frame structures, which are widely utilized in the fields of structural engineering, e.g. large stadiums, bridges, and radio towers. The lightness and slenderness of frame structures are the favorable features that can contribute to resource saving and eco-friendly. However, these features are in a lack of stiffness, which may lead to vibration problems. Hence, the optimal design by considering the dynamic characteristics is essential and basically required for designing frame structures. On the other hand, the natural frequencies, i.e., vibration eigenvalues, usually represent the dynamic characteristics of structures. Especially the lower order natural frequencies are considered as an evaluation of the dynamic stability of structures. The dynamic response of structures can be substantially reduced by enhancing their fundamental frequencies [1–4]. Therefore, fundamental frequency can be considered as a significant design criterion in practical engineering applications of frame structures.

In terms of maximization of natural frequencies, based on finite element method (FEM), extensive studies have been carried out to propose various shape and topology design optimization methods for continua and shell structures over the past several decades [5–13]. As a shape optimization method, Olhoff [5] introduced a cross-sectional shape optimization of transversely vibrating beams for achieving the maximization of natural frequencies. Narita and Robinson [6] proposed a method for determining the maximum fundamental frequencies of laminated panels. Based on the traction method, Liu and Shimoda [7] presented a non-parametric shape optimization method for natural vibration design of stiffened thin-walled or shell structures. Using a topology optimization method, Diaz and Kikuchi [8] reported a homogenization based method to optimize the reinforcement of given 2D structures to maximize their natural frequencies. Pedersen [9] showed a topology optimization for optimizing the natural frequencies of plates based on the SIMP method. Using the level-set based

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http://dx.doi.org/10.1016/j.ijmecsci.2016.08.024
0020-7403/

topology optimization method, Osher and Santosa [10] studied the maximization of natural frequency and spectral gap of a two-material inhomogeneous drum. Allaire and Jouve [11] presented a level-set topology optimization method based on a combination of the classical shape derivative and the Osher-Sethian level-set algorithm to maximize the fundamental eigenvalue. Lee and Lee [12] proposed a numerical method for the optimal design of cable-strut structures using vibration analysis. However, there are few studies contribute to the optimization of natural frequencies of frame structures, which is extremely important for applications of the frame structures in urban architecture, civil engineering, and so on.

According to the optimization of frame structures, similarly to continua and shell structures, it can be commonly classified into three categories, which are size optimization, topology optimization and shape optimization [14]. In the early stage of the optimal design of frame structures, the topology was often treated as the design variables, and in that case, the mechanical characteristics can be drastically improved rather than optimizing the shape or size of cross-section. In the past decades, some optimization methods for frame structures have been proposed [15–26]. For instance, Wang et al. [15] reported simultaneous shape and sizing optimization to minimize the weight under multiple frequency constraints by calculating the integrated discrete sensitivity numbers. Sedaghati and Esmailzadeh [16] developed a new optimization algorithm to determine the minimum weight of frame structures based on the sequential quadratic programming (SQP) method. Ohmori and Yamamoto [17] also applied the SQP method to grid and lattice shells. A number of heuristic approaches have also been reported. Hashemian et al. [18] applied the genetic algorithm (GA) method for latticed structures under a compressive axial load to achieve a maximum buckling load. The GA method has been widely applied to shape, size or topology optimization of trusses or grid shells [19–22], but only can be used in designing structures with small number of members and degrees of freedom. In addition, Kaveh and Talatahari [23] reported an optimal design of space trusses for achieving the minimum weight subjected to the stress limitations using the Cuckoo search algorithm with Levy flights. Hamza et al. [24] applied the taboo search to optimization design of a class of plane trusses. Rial [25] used the simulated annealing for shape optimization of one layer lattice shells. In addition, Kamiński and Solecka [26] optimized the truss-type structures using the generalized perturbation-based Stochastic Finite Element Method. However, most of those proposed methods are classified as parametric approaches. Because the obtainable shapes strongly depend on the parameterization processes, the parametric optimization design requires preliminary shape parameterization of basis models and sufficient knowledge of designers. Except for topology optimization, there is no non-parametric shape optimization method of frame structures in terms of natural frequency problem as far as we know.

Considering the huge number of members and degrees of freedom within a frame structure, a non-parametric optimization method that can efficiently solve a large scale problem has much utility value. The free-form optimization method adopted in the present work is a kind of non-parametric optimization method, and is also called the H^1 gradient method. The H^1 gradient method was firstly proposed by Azegami [27] and Shimoda et al. [28–31] developed this optimization method for designing shell structures, membrane structures, and composite structures. Furthermore, this method was also developed for determining the optimal free-form of frame structures in terms of compliance minimization problem [32]. In this study, we aim to extend it for the application in the natural frequency maximization problem. The free-form optimization method involves two key techniques, which are (1) theoretical derivation and numerical calculation of the shape gradient functions for a design problem, and (2) adopting the gradient method in the Hilbert space with a P.D.E (Partial Differential Equation) smoother for decreasing the objective functional while maintaining the mesh regularity, which refers to the optimal shape.

In the fundamental frequency maximization problem considered in the present work, we need to solve the repeated eigenvalue problem. Actually, vibration analysis and buckling analysis frequently encounter the repeated eigenvalue problem [33]. In shape design optimization, the sensitivity function (e.g. shape gradient function) of a single eigenvalue can be derived from material derivative, while the sensitivity function of the repeated eigenvalue problem with r-fold repeated eigenvalues should be calculated from an $r \times r$ matrix by employing directional derivative [13]. Considering repeated eigenvalues in the maximization of fundamental frequency problem, the directional derivative of the fundamental eigenvalue is affected by the neighbor eigenvalues, which often leads to a vibration or a divergence of the objective functional during the iteration history. In other words, the discontinuity of repeated eigenvalues causes separation or combination of the repeated eigenvalues. Hence, it is essential to solve the repeated eigenvalue problem for the stable convergence of the objective functional. Ma et al. [1] proposed a new weighted objective functional involving all the eigenvalues to theoretically improve the discontinuity of repeated eigenvalues, which could sufficiently deal with several different eigenfrequency optimization problems. Xia et al. [13] presented a level-set based shape and topology optimization method by taking different approaches to derive the boundary variation to maximize the simple or 2-fold repeated first eigenvalues of structure vibration, in which they devised the sensitivity of repeated eigenvalues by setting the off-diagonal terms of the 2×2 matrix to be zero for the ascent direction of boundary shape. In the present work, we consider the eigenvalues in a given range nearby the fundamental frequency as repeated eigenvalues, and set a simple switching technique of using the sum of objective functional for overcoming the discontinuity problem of repeated eigenvalues. This method can simply calculate the sensitivity function to increase the sum of repeated eigenvalues.

In the following sections, the governing equation of frame structures and the formulation of the non-parametric shape optimization problem for maximizing the fundamental frequency are described. The optimality conditions and the shape gradient functions are then theoretically derived for the fundamental frequency maximization problem. Next, based on the developed H^1 gradient method, we construct a free-form optimization system for frame structures to maximize their fundamental frequencies considering repeated eigenvalues. Finally, three numerical examples are carried out to confirm the effectiveness and validity of the proposed method.

2. Non-parametric shape optimization problem of frame structures

2.1. Governing equation of frame structures

As shown in Fig. 1, a frame structure is defined as an assembly of arbitrarily curved members $\left\{\Omega^j\right\}_{j=,2,\ldots,N}$, where N represents the total number of members. The frame structure occupies a bounded domain ($\Omega = \{\Omega^j\}_{j=,2,\ldots,N} \subset \mathbb{R}^3$), and

$$\Omega^j = \left\{ \left(x_1^j, x_2^j, x_3^j\right) \in \mathbb{R}^3 \middle| \left(x_1^j, x_2^j\right) \in A^j \subset \mathbb{R}^2, x_3^j \in S^j \subset \mathbb{R} \right\},$$
$$\Gamma^j = \partial A^j \times S^j, \quad \Omega^j = A^j \times S^j \tag{1}$$