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A fractal approach of the sound absorption behaviour of materials. Theoretical and experimental aspects

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ABSTRACT

In the materials area there are many theoretical and experimental investigations concerning their sound absorption behaviour, covering a wide range of applications. An alternative approach consists in identifying the ensemble acoustic field-propagation environment with a fractal gives its functionality in the form of structure parameters that is dependent on scale resolution. These fractal parameters can be matched with "classical" ones typical for sound absorption experiments in various materials. The mathematical methodology presented here implies the substitution of a dynamics with constraints on continuous but differentiable curves, in an Euclidian space, with "synchronous" dynamics, free from any constraints, on continuous but non-differentiable curves with various fractal dimensions but on a fractal space (i.e. the geodesics of that space). In a very special representation, the external constraints select the fractal geodesics type. In our study this "type selection" refers to the geodesics tunnel effect of fractal (acoustic) type for which we calculate the reflectance and the transparency of an external fractal barrier. An experimental procedure, using a modified impedance tube technique, to determine the sound absorption coefficients for various composite materials was conducted. The procedure uses an anechoic room and the measured sound absorption coefficients also include the sound transmission. The fractal approach of the acoustic behaviour, through the fractal parameters determined (transparency and reflectance) matched the experimental results, in terms of sound absorption, emphasizing the high degree of generality of the fractal theory in the dynamics of physical processes.

1. General considerations. From differentiability to non-differentiability in the acoustic process dynamics

Acoustic behaviour of materials remains a subject of great interest due to many applications involving the requirement to attenuate or isolate industrial and architectural components sound. In our approach, a material can be modelled by accessing some dynamics on non-differentiable curves with various fractal dimensions. The ensemble acoustic field-propagation environment can be assimilated to a complex system, if we take into account both their functionality, as well as their structure [1,2]. We will call it fractal acoustic system, as it was extensively presented in Appendix. The models commonly used to study the dynamics of complex systems are based on the assumption, otherwise unjustified, of the differentiability of the physical variables that describe them (for example, density, momentum, energy etc.). The success of differentiable models must be understood sequentially. i.e. on domains large enough for the differentiability and integrability to be valid.

But differential method fails when facing the physical reality of complex systems dynamics. In order to describe such physical dynamics of complex systems, and still remaining tributary to differential hypothesis, it is necessary to introduce, in an explicit manner, the scale resolution in the expressions of the physical variables that describe these dynamics and, implicitly, in the fundamental equations of evolution (for example, density, momentum and energy equations). This means that any dynamic variable, dependent, in a classical meaning, on both spatial and temporal coordinates, becomes dependent also on the scale resolution, in this new context [3–5]. In other words, instead of working with a dynamic variable, described through a strictly non-differentiable mathematical function, we will just work with different approximations

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on that function, derived through its averaging at different scales resolution. Consequently, any dynamic variable acts as the limit of a functions family, the functions being non-differentiable for a null scale resolution and differentiable for a nonzero scale resolution.

The non-differential approach is well adapted for the field of complex systems, where any real determination is conducted at a finite scale resolution. This implies the development of a new physical theory applied to complex systems for which the motion laws, invariant to spatial and temporal coordinates transformations, are integrated with scale laws, invariant at scale transformations. Such a theory based on the above presented assumptions was first developed in the Scale Relativity Theory [3] with fractal dimension 2 and more recently, in the Scale Relativity Theory with an arbitrary constant fractal dimension [4,5]. Both theories define the "fractal physics models".

The fractal physics models consider that the dynamics of complex system structural units take place on continuous but non-differentiable curves (fractal curves). In such context, constraint dependent dynamics, in an Euclidian space (on continuous but differentiable curves) are substituted by constraint independent dynamics in a fractal space (on continuous but non-differentiable curves-fractal geodesics). Any other external constraint will be understood as a selection procedure of the fractal geodesics in the fractal space. Thus, all structural units of the complex systems are substituted with their respective geodesics. Moreover, for time scales large with respect to the inverse of the maximum Lyapunov exponent [6,7], deterministic trajectories can be replaced by families of potential trajectories, i.e. fractal geodesics, and the concept of defined positions by that of probability densities.

In the following, let us explain the above mentioned "methodology" used to describe a fractal acoustic system. The functionality of such dynamics can be sustained by means of the collision processes (in the form of acoustic pressure). Between two successive collisions (either phononic collisions as in the solid-state case, or particles collisions as in the fluid state, etc.), the trajectory of any fractal acoustic system entity is a straight line that becomes non-differentiable at the impact point. Considering that all collisions impact points form an uncountable set of points, it results that trajectories of all fractal acoustic system entities become continuous but non-differentiable curves. Once specified the curves type, in a fractal space they will be identified, through the motion equations, with its geodesics (fractal geodesics). On a fractal space can simultaneously "operate" various non-differentiable dynamics: of quantum type in the fractal dimension $D_F = 2$, of correlative type in fractal dimension $D_F < 2$ or dynamics of non-correlative type in fractal dimension $D_F > 2$ (details in [3,6]). As consequence, on such a space it can coexist non-differentiable motion curves (fractal curves) with various fractal dimensions. This particularity implies a multi-fractality in the overall behaviour of the system. The dynamics selection, and so the selection of fractal curves "classes" still remains tributary to the external constraints (shape of source and propagation environment structure, experiment geometry, etc.). Let us note that in our case the external constraint was associated with a rectangular barrier of potential.

Now, the mathematical procedure implies the following steps:

(i) obtaining of the fractal geodesics equations;

(ii) finding of the fractal geodesics equations solutions, based on "adequate" initial and boundary conditions imposed by external constraints;

(iii) the parameters "generation" by means of solutions of the fractal geodesics equations, which are in correspondence with some data provided by experiments.

Moreover, in this work, we implemented the previous "steps" and go through the following stages in agreement with the mathematical procedure above mentioned:

(i) The fractal geodesics were obtained in the hypothesis of external constraints equivalent to one-dimensional (acoustic) potential barrier of rectangular shape. Thus, the problem is reduced to a stationary dynamics one, considering the tunnel effect of fractal (acoustic) type;

(ii) The stationary solutions of the tunnel effect of fractal (acoustic) type were obtained by imposing "adequate" initial and boundary conditions;

(iii) First at all, the fractal (acoustic) reflection factor and the fractal (acoustic) transmission factor were built. Further, the fractal (acoustic) reflectance and the fractal (acoustic) transparency were then obtained. These last two parameters were put in correspondence with directly measurable parameters, as the sound absorption coefficient obtained in a proposed experimental procedure.

2. Mathematical aspects. Dynamics by means of tunnel effect of fractal (acoustic) type

According to the aforementioned statements, hereinafter we can consider a fractal acoustic system whose fractal (acoustic) entities are moving on continuous but non-differentiable curves (fractal curves). In such conjecture, the fractal acoustic system dynamics are described through the fractal geodesics of following form (see Eq. (A.26) from Appendix):

$$\lambda^2 (dt)^{(4/D_F)-2} \partial^l \partial_l \Psi + i\lambda (dt)^{(2/D_F)-1} \partial_t \Psi - \frac{U}{2}\Psi = 0 \tag{1}$$

where: $\partial_l = \frac{\partial}{\partial x_l}, \quad \partial^l \partial_l = \frac{\partial^2}{\partial x_l \partial x^l}, \partial t = \frac{\partial}{\partial t}, \quad l = 1,2,3$

In Eq. (1): Ψ is the fractal (acoustic) state, x^l are the fractal spatial coordinates, U is the external scalar potential, λ is the specific coefficient associated to fractal–nonfractal transition, dt is the scale resolution of acoustic type and D_F is the fractal dimension of a motion curve of the fractal acoustic system entity. Details on how to obtain Eq. (1), the meanings of the operating variables from this equation are exhaustive presented in Appendix. Thus, for the one-dimensional case Eq. (1) can be rewritten as:

$$\lambda^2 (dt)^{(4/D_F)-2} \partial_{xx} \Psi(x,t) + i\lambda (dt)^{(2/D_F)-1} \partial_t \Psi(x,t) - \frac{U}{2}\Psi(x,t) = 0 \tag{2}$$

If the external scalar potential U is time independent, $\partial_t U = 0$, Eq. (2) admits the fractal stationary solution:

$$\psi(x,t) = \theta(x) \exp\left[-\frac{i}{2m_0 \lambda (dt)^{(2/D_F)-1}} Et \right] \tag{3}$$

where E is the fractal (acoustic) energy and m_0 is the rest mass respectively of the fractal (acoustic) entity. Then θ(x) becomes a fractal solution of the fractal non-temporal equation:

$$\partial_{xx}\theta(x) + \frac{1}{2m_0 \lambda^2 (dt)^{(4/D_F)-2}} (E-U)\theta(x) = 0 \tag{4}$$

Now, we can describe, through Eq. (4), the dynamics of the acoustic complex field (see Eq. (A.24) from Appendix) in the form of fractal (acoustic) states Ψ, when Ψ, suffers" constraints given by the following external scalar potential configuration (Fig. 1). It has been selected the simplest potential configuration in the form of fractal (acoustic) barrier.

The external scalar potential is described through Eq. (5):

$$U(x) = \begin{cases} 0 & -\infty < x < 0 \\ U_0 & 0 < x < a \\ 0 & a < x < +\infty \end{cases} \tag{5}$$

where U_0 is the fractal (acoustic) barrier height and a is its width. These acoustic dynamics can be perceived, as "functionality" of a tunnel effect of fractal-acoustic type. A fractal-acoustic entity with known energy penetrates a barrier of greater energy than the incident one (in conditions in which the entity is identified with its own geodesic).

As it is shown in Fig. 1, the fractal real straight line $\{x/x \in R\}$ is structured in three regions, denoted by 1, 2, 3 and called, of fractal-acoustic incidence, of fractal-acoustic barrier and of fractal-acoustic emergence, respectively. The energy E of the entity of the fractal-acoustic system dynamics was deliberately chosen smaller than U_0, in