International Journal of Non-Linear Mechanics 67 (2014) 7–15

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

Bifurcation analysis of an electro-statically actuated micro-beam in the presence of centrifugal forces

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ARTICLE INFO

Article history:
Received 26 February 2013
Received in revised form
4 July 2014
Accepted 6 July 2014
Available online 22 July 2014

Keywords:
Centrifugal force
Pitch-fork bifurcation
Saddle-node bifurcation
Pull-in angular velocity

ABSTRACT

In this paper, the influence of centrifugal forces on the stability of an electro-statically actuated clamped–clamped micro-beam has been investigated. The non-dimensional governing static and dynamic equations have been linearized using the step by step linearization method (SSLM), then, a Galerkin-based reduced order model has been used to solve the linearized equations. For constant value of a bias DC voltage and different values of angular velocity the equilibrium points of the corresponding autonomous system including stable center points, unstable saddle points and singular points have been obtained using the equivalent mass-spring model. Subsequently the bifurcation diagram has been depicted using the obtained fixed point. The static pull-in voltage value for different values of angular velocity and the static pull-in angular velocity for different values of bias voltage have been calculated. The obtained results are validated using results of previous studies and a good agreement has been observed. The effect of the centrifugal force on the fixed points has been studied using the phase portraits of the system for different initial conditions. Moreover, the effects of centrifugal forces on the dynamic pull-in behavior have been investigated using time histories and phase portraits for different angular velocities.

1. Introduction

MEMS are devices and systems of distinguished properties and unique characteristics. Micro-electro-mechanical systems (MEMS) is a technology that in its most general form can be defined as miniaturized mechanical and electro-mechanical elements, devices and structures that are made using the micro-fabrication techniques [1]. MEMS technology has emerged to create systems on the sub-micrometer scale, the geometric dimensions of the MEMS devices can be from well below one micron to several millimeters.

Commonly used MEMS devices are miniaturized structures, sensors, actuators, and microelectronics, the most notable and perhaps most interesting devices are the microsensors [2,3] and micro-actuators [4]. Microsensors and micro-actuators are appropriately categorized as "transducers", which are devices that convert energy from one form to another. In the case of microsensors, the device typically converts a measured mechanical signal into an electrical signal. In the most basic form, the sensors gather information from the environment through measuring mechanical, thermal, biological, chemical, optical, and magnetic phenomena; the electronics process the information derived from the sensors then direct the actuators to respond by moving, positioning, regulating, pumping, and filtering, in order to control the environment for some desired outcome or purpose.

Nowadays, with the development of microsystems technology, many micro-electro-mechanical systems (MEMS) such as microswitches [5,6], resonators [7,8], micromirrors [9], microphones [10], nano-tweezers [11] and etc. are widely designed, analyzed, fabricated and used.

Electrostatic transduction is the most common actuation and sensing method in MEMS because of its simplicity, favorable scaling property, lower driving power, large deflection capacity, relative ease of fabrication, and better performance than other excitations. Most of MEMS systems, which work with electrostatic actuation, include a movable part such as beams, plates, membranes, and other mechanical components. To design and calculate the operating condition of these devices, the main challenges that face MEMS engineers in modeling and simulating the static and dynamic behavior of these systems under different conditions.

When the applied voltage to the electro-statically actuated micro-beam reaches a critical value the saddle node bifurcation instability occurs [12], this phenomenon is called pull-in instability in MEMS literature and the corresponding voltage is called pull-in voltage. First time Nathanson et al. [13] observed pull-in phenomenon experimentally and also they exposed and analyzed a mass-spring model of electrostatic actuation and provided the first theoretical description of pull-in instability. Zhang and Zhao [12] investigated the pull-in instability of an

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http://dx.doi.org/10.1016/j.ijnonlinmec.2014.07.001
0020-7462/

electro-statically actuated MEMS switch. Kacem et al. [14,15] described a comprehensive nonlinear multi-physics model and the critical amplitude and the pull-in instability of doubly clamped nano-beam as well as NEMS cantilevers which applied to both mechanical and electrostatic nonlinearities. Younis et al. [16] studied the influence of combined mechanical shock and electrostatic forces on the pull-in instability of micro-beams; they used a reduced-order model to predict the responses and the obtained results are verified by finite-element simulations. Rezazadeh et al. [17] studied the static and dynamic responses of micro-beams using both the lumped and the distributed models to a DC and a step DC voltage. Vakili-Tahami et al. [18] considered the static and dynamic pull-in phenomenon of a capacitive nano-beam, using Euler–Bernoulli beam theory. Najar et al. [19] investigated the dynamic behavior of a micro-beam based electrostatic micro-actuator and studied the influence of varying geometry of the micro-beam on the dynamic response of electrostatic micro-actuators. Yagubizade et al. [20] presented the effects of squeeze-film damping on the dynamic response of electro-statically-actuated clamped–clamped micro-beams. Kacem et al. [21] presented the dynamic behavior of clamped–clamped electro-statically actuated nano-resonators and proposed a way to retard the pull-in by decreasing the AC voltage in order to enhance the performance of NEMS resonators.

Bifurcation is a French word, first time it was introduced into nonlinear dynamics by Poincare to indicate a qualitative change in the features of a system, such as the number and type of solutions, under the variation of one or more parameters on which the considered system depends [22]. In the case of bifurcation behavior of MEM and NEM systems some works have been carried out up to now. Kacem and Hentz [23] reported the experimental observation of a four-bifurcation-point behavior of electro-statically actuated micromechanical resonators; they also demonstrated both analytically and experimentally tuning the bifurcation topology of this behavior via an electrostatic mechanism. In other work Kacem et al. [24] investigated stability control of nonlinear micromechanical resonators, and they showed how the bifurcation topology of an undesirable unstable behavior is modified when the resonator is simultaneously actuated at its primary and superharmonic resonances. Lin and Zhao [25], studied the dynamic behavior of nano-scale electrostatic parallel-plate RF switches with the consideration of the van der Waals (VdW) effects, and analyzed the bifurcation phenomena of them. In other paper they [26] have studied stability and bifurcation behavior of electrostatic torsional NEM varactor and also they used mass-spring model for their researches. Guo and Zhao [27] analyzed the influence of VdW force on the stability of electrostatic torsional nano-electro-mechanical systems (NEMS) actuators. Chao et al. [28] used the techniques of bifurcation and phase portrait analyses for a generalized clamped–clamped micro-beam to predict the DC dynamic and static pull-in voltages. Rezazadeh et al. [29] studied the static and dynamic stabilities of a micro-beam with various boundary conditions actuated by a DC piezoelectric voltage. Abbasnejad and Rezazadeh [30] studied the mechanical behavior of cantilever and fixed–fixed FGM micro-beams subjected to a nonlinear electrostatic pressure. In recent work Mobki et al. [31] studied the mechanical and bifurcation behavior of a capacitive micro-beam suspended between two conductive stationary plates.

Centrifugation is the force which describes the outward pressure that is exhibited around an object rotating around a central point. The centrifugal force plays a major role in many applications, such as, centrifugal turbo-machinery [32], testing the failure rates of micro-mechanical structures [33], non-contact batch micro-assembly [34], micromanipulation technique to design and build structures and devices [35], and centrifuge device to spins blood samples to separate the solids from the liquids [36], etc. Approximately all of the rotating machinery requires to be measured and adjusted the angular velocity [37], to this end a micro-beam or another micro-structure element can be used as an angular-rate sensors [38] or spinning-mass gyroscopes [39].

In spite of existence of lots of researches concerning the mechanical behavior and the stability of MEM structures, there is not any extensive study describing the stability of these structures from bifurcation point of view. Therefore, in this paper a clamped–clamped micro-beam suspended between two conductive substrates is considered. Because of applying a bias DC voltage, the micro-beam is subjected to an electrostatic force. Also the micro-beam is in the presence of a centrifugal force acting in transversal direction. In order to study the static and dynamic behavior of the micro-beam the governing equation is transformed into non-dimensional form. Then, the step by step linearization method (SSLM) and Galerkin-based reduced order model have been used to solve the dimensionless governing equation. Equilibrium positions of the system for different values of angular velocity and constant bias voltage are calculated using lumped mass-spring model; then, bifurcation diagram is plotted and for some values of angular velocity stability of the fixed points is investigated in detail using the phase portraits at the center of the micro-beam. Also, it can be stated that, the provided system can be utilized as a sensor for measuring of the angular velocity [37].

2. Model description and assumptions

As shown in Fig. 1 the proposed micro-structure model consists of a clamped–clamped micro-beam suspended between two stationary electrodes which are subjected to electrostatic and centrifugal forces. The microstructure is mounted on a rotating machine and the length of the beam is L, width b, thickness h, and the radius of the rotating machine is r.

In order to simplify the analysis of bifurcation behavior of micro-beam, the illustrated model in Fig. 1 is replaced with an equivalent mass-spring model, which is shown in Fig. 2.

3. Mathematical modeling

The electrostatic force, applied by a bias DC voltage, in this model can be introduced using a standard capacitance model [40]

$$P(V,w)=\frac{1}{2}\frac{\varepsilon_0 bV^2}{(g_0-w(x,t))^2}-\frac{1}{2}\frac{\varepsilon_0 bV^2}{(g_0+w(x,t))^2} \tag{1}$$

where ε_0 and g_0 are the dielectric constant of the gap medium and initial gap, respectively. Furthermore, V is the applied bias voltage to the upper and lower substrates, and w is the flexural deflection of the micro-beam. The centrifugal force applied to the micro-beam, caused by rotation of a rotary machine, is presented by the following equation:

$$q(w,\omega)=\rho A(r+g_0-w(x,t))\omega^2 \tag{2}$$

where

$$F(V,w,\omega)=\frac{1}{2}\frac{\varepsilon_0 bV^2}{(g_0-w(x,t))^2}-\frac{1}{2}\frac{\varepsilon_0 bV^2}{(g_0+w(x,t))^2}+\rho A(r+g_0-w(x,t))\omega^2 \tag{3}$$

The governing equation for the dynamic behavior of a micro-beam based on Euler Bernoulli theory actuated by an electrostatic force is as follows [41]:

$$\tilde{E}I\frac{\partial^4 w}{\partial x^4}-f\frac{\partial^2 w}{\partial x^2}+\rho A\frac{\partial^2 w}{\partial t^2}+c\frac{\partial w}{\partial t}=F(V,w,\omega) \tag{4}$$

Where I is the moment of inertia of the cross-section, ρ is the density, and r is the radius of a shaft micro-beam installed on it and f is the stretching force and is created when a fixed–fixed