International Journal of Non-Linear Mechanics 67 (2014) 218–230

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

Nonlinear spatio-temporal instability regime for electrically forced viscous jets

Saulo Orizaga [a,*], Daniel N. Riahi [b], L. Steven Hou [a]

[a] Department of Mathematics, Iowa State University, 2229 Lincoln Way, Ames, IA 50011, USA
[b] Department of Mathematics, University of Texas Pan-American, Edinburg, TX 78539, USA

ARTICLE INFO

Article history:
Received 22 July 2014
Received in revised form
3 September 2014
Accepted 3 September 2014
Available online 17 September 2014

Keywords:
Wave interaction
Resonant instability
Spatial instability
Jet flow
Electric field
Temporal instability

ABSTRACT

This paper considers the problem of nonlinear instability in electrically driven viscous axisymmetric jets with respect to spatial and temporal growing disturbances in the presence of a uniform or non-uniform applied electric field. The mathematical modeling for the jets, which uses the original electrohydrodynamics equations (Melcher and Taylor, 1969) [8], is based on the nonlinear mechanics that govern the liquid jet due to tangential electric field effects. At the linear stage, we found that a particular jet of fluid could exhibit the Rayleigh and Conducting flow Instabilities for the spatial and temporal evolution of the disturbance. For the nonlinear regime of the problem, we studied the resonant instability and nonlinear wave interactions of certain modes that satisfy the dyad resonant condition. The nonlinear wave interactions in the jet provided a significant change in the fluid flow properties that extend notably the available understanding of the problem at the linear stage. It was found that the nonlinear resonant instability provides an amplifying effect on the magnitude of the disturbances which evolves the jet to reduce significantly its radius at a shorter axial location. For the case of higher viscosity fluid, the electric field in the jet was found to be increasing spatially and temporally when nonlinear wave interactions were taken into account during the resonant instability. The resulting nonlinear solutions for the jet thickness, jet's electric field, jet's surface charge and jet velocity are presented and discussed.

1. Introduction

1.1. Relevant investigations and applications

The investigations of electrically driven jets and their instabilities have attracted many investigators and many efforts have been made to understand the physical laws relating to liquid properties and flow rate to the size and charge of the emitted thin viscous jet [7,8]. Electrically driven jets are found to have applications in electrospraying [11] and electrospinning [6,9,10]. Electrospraying is a process that is aimed to produce very small and uniform droplets by the use of electric fields. Electrospinning is a process for manufacturing high volumes of very thin fibers that typically range from 100 nm to 1 μm, with lengths up to several hundred of meters depending on the application, from a vast variety of materials, including polymers, composites and ceramics [3,5,20]. In this process, nanofibers are produced by solidification of a polymer solution stretched by an electric field. The unique properties of nanofibers are extraordinarily high surface area per unit mass, very high porosity, tunable pore size and surface properties, layer thinness, high permeability, low basic weight, ability to retain electrostatic charges and cost effectiveness. These electrospun nanofibers have many practical applications to different areas including wound dressing, drug or gene delivery vehicles, high quality filters, biosensors, fuel cell membranes and electronics, tissue-engineering processes. The motivation for this investigation is to contribute in the understanding of electrically driven jets [2,4,15,16,18], which at the application level this problem is still performed on a trial and error basis due to the complexity and nonlinear nature of the problem.

In this paper, we consider the non-linear problem of the axisymmetric electrically driven jets with finite electrical conductivity and under the presence of a uniform and non-uniform applied electric field [9,10,19,21], but now we include realistic features that are found naturally in the jet flow including time and space evolving instabilities which are known to exist [8]. We approach the non-linear problem by considering combined space and time dyad resonant instability modes to study the non-linear wave interaction of such growing disturbances. Resonant wave interactions play a very important role in the theory of weakly non-linear instability of weakly non-linear waves. In general terms, understanding the small amplitude solutions of various physical problems

* Corresponding author. Tel.: +1 515 294 1752; fax: +1 515 294 5454.
E-mail addresses: sorizaga@iastate.edu (S. Orizaga), driahi@utpa.edu (D.N. Riahi), stevehou@iastate.edu (L. Steven Hou).

http://dx.doi.org/10.1016/j.ijnonlinmec.2014.09.001
0020-7462/

requires a first natural step, which is the linearization of the equation or system of equations. Then by the use of harmonic analysis, one can apply the principle of superposition to provide a solution representation of the linearized problem in the form of plane waves

$$A \exp[i(\mathbf{k}\cdot\mathbf{x}-\omega t)] \tag{1}$$

where ω is the frequency of the wave, $\mathbf{k}$ is a wave number vector, whose magnitude is referred to as wave number, and A is the amplitude of the wave. The wave number and the frequency are then involved mostly in the resulting dispersion relation from the linear system derived from particular physical problem. In the study of weakly non-linear wave interactions solutions, we look for the solutions in the form of plane waves as we described above by working with the linearized system, but the amplitudes of the plane waves will no longer be constant. Instead, the amplitudes are slowly evolving or modulated by non-linear wave interactions, which occur due to the non-linearities that exist on the original equations and boundary conditions of the system.

The connection of resonant instability and non-linear wave interaction relies on the behavior that a system can undergo with regard to large amplitude oscillations at certain frequencies. The frequencies that generate larger oscillation amplitudes compared to all other frequencies are called resonant frequencies. These frequencies are very critical because a system response could be altered even by small periodic driving forces [1,14]. In general, the system could be stabilized or destabilized by such driving forces, which for our problem investigation these take the form of the non-linearities in the original system of partial differential equations. The general ideas of resonance wave interactions have been attributed to several authors such as H.J. Beth and Diederik Korteweg [13], but more recent work has been done by Rott [1]. He investigated the internal resonance for the double pendulum problem, which was modeled by a system of ordinary differential equations that described the dynamics of the angles of rotation θ_1 and θ_2 in the system. He provided the results, which are now implemented in several areas of research, relating the small oscillations normal modes of the system and the corresponding modifications of such modes due to the non-linearities of θ_1 and θ_2. Rott [1] showed that these non-linearities treated as forcing terms in the system are the weak interactions of the normal modes. He found that there is a slow periodic interchange of energy between the two normal modes. The resonance he investigated was of dyad (two-wave) type, which is the type of resonance we study for this investigation by mathematically adapting those ideas to a larger system and far more complicated mathematical model which consists of four non-linear partial differential equations.

In regards to the resonant wave interactions in electrically driven jets, no investigation had been carried out for electrically driven jets prior to the work of Orizaga and Riahi [21,23]. They conducted the investigation of temporal linear instability of the modes that satisfy the dyad resonance conditions. They found that the instabilities of resonant type evolving in time were able to produce both favorable and unfavorable results. The unfavorable condition was in the sense that the instabilities detected on the linear case were actually modified by stronger types of instabilities for most cases but at the same time this in turn provided a significant reduction on the jet radius, which is of high interest for practical applications.

In this investigation in contrast with [21], we considered a more realistic study, which included two types of instabilities in a combined form for space and time. We modeled the combined spatial and temporal evolving instabilities for axisymmetric electrically driven jets, which are known to exist by the experiments done in [7,8,10]. We extended the linear studies done in [9,10,19] by retaining the nonlinearities governing the jet flow. We found, in particular, that for certain parameter values of the jet flow system, there are some resonance modes that can dominate the jet for its temporal and spatial evolution. We were able to solve for the nonlinear quantities in the jet for our investigation and this allowed a favorable change in the dynamics of the jet flow. The significant changes due to nonlinear interactions were characterized by changing a thickening to a thinning jet along with increasing the electric field driving the jet. Our study was able to uncover new parameter regimes for both spatial and time modes in which instabilities were significantly enhanced and jet radius was reduced, providing a mechanism of importance in applications. By considering resonant instability and nonlinear wave interactions in electrically driven jets (i.e. considering the nonlinear problem without linearizing), we were able to explore and understand more about the mathematical model and its properties that were previously studied only at the linear stage [9,10,19].

1.2. Paper organization

The structure of the paper is as follows. In Section 2, we present the mathematical model for the axisymmetric electrically driven jets. The jet flow is studied in the classical linear stability approach, and the nonlinear instability is studied in the dyad resonance sense. The dispersion relation governing the jet flow is developed and its solution allows for the detection of the Rayleigh and Conducting Instabilities. For the nonlinear problem, we model the disturbances as wave packets and with the nonlinear wave theory we arrive at the system of partial differential equations governing the amplitude of such disturbances. The system of PDEs is solved for the amplitude functions, which carry the nonlinear wave interactions that evolve in both time and space. In Section 3, we provide the main results, simulations and discussions for this investigation. In Section 4, we provide some concluding remarks.

2. Electrically driven jets modeling and nonlinear wave theory

2.1. Mathematical formulation

Our mathematical modeling of the electrically driven jets is based on the governing electrohydrodynamic equations [8] for the mass conservation, momentum, charge conservation and for the electric potential, which are described in [9,10,19,21]. The work of dyad resonant combined time and spatial instability that we consider in this paper is a research continuation of the work done in [21]. In contrast to the time evolving instability that was studied in [21], here we study particular realistic features of spatial and temporal instabilities of the modes that enhance significantly in space and time as observed in experiments [10]. We find that combined space and time evolving instabilities subjected to dyad resonance conditions can be significantly larger in magnitude than those corresponding to temporal instability modes [21]. The instabilities in the present study can occur at a very short distance after the jet is emitted which agrees with the experiment results in [7,8]

$$\frac{D\rho}{Dt}+\rho\nabla\cdot\vec{u}=0 \tag{1a}$$

$$\rho\frac{D\vec{u}}{Dt}=-\nabla P+\nabla\cdot\nabla(\mu\vec{u})+q\vec{E} \tag{1b}$$

$$\frac{Dq}{Dt}+\nabla\cdot(K\vec{E})=0 \tag{1c}$$

$$\vec{E}=-\nabla\Phi \tag{1d}$$

where $D/Dt=\partial/\partial t+\vec{u}\cdot\nabla$ is the total derivative, $\vec{u}$ is the velocity vector, P is the pressure, $\vec{E}$ is the electric field vector, Φ is the electric