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Comparison of a composite model and an individually fiber and matrix discretized model for kink band formation

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Failure by kink band instability in a unidirectional fiber composite is analyzed. A micromechanical discretized finite element model is used and compared to an existing composite constitutive model. The comparison is made in a fiber angle vs. applied compressive stress space. An investigation on the relation between the kink band angle and the fiber angle is conducted in the postbuckling regime. The critical kink band angle is examined for different initial fiber misalignment angles.

1. Introduction

Unidirectional fiber composites subjected to compressive stress may fail by several different modes [1]. One of the main failure modes is plastic microbuckling which leads to a kink band instability. At kinking, a narrow band is formed into which strains localize. The phenomenon is seen in different anisotropic materials, and was observed in phyllite by Paterson and Weiss [2]. In the early work on fiber kinking, models were formulated treating the fibers as beams on an elastic foundation. Rosen's [3] model leads to the critical compressive stress for kink band formation being equal to the shear modulus of the matrix. Later Argon [4] formulated a model using the shearing yield stress of the matrix and the fiber misalignment to determine the critical stress. Budiansky [5] incorporated the effect of an elastic ideally plastic matrix containing previous results as special cases.

To analyze kink bands, Christoffersen and Jensen [6] developed a rate constitutive equation accounting for the microstructure of a unidirectional fiber composite. They treated the problem in the framework of localization of deformation [7]. The model allowed for arbitrary elastic–plastic behavior of the constituents, and it will be reviewed briefly later in the paper. The model was applied in a study of initial fiber misalignments [8] and solutions were obtained by a numerical scheme by increasing the fiber angle incrementally and satisfying equilibrium.

Recently Wadee et al. [9] developed a geometrical kink band model founded on potential energy principles. It was further developed by Zidek and Völlmecke [10] to include non-linear material behavior of the matrix.

Another way of attacking the kink band problem is by making an individual fiber and matrix discretized finite element analysis. This was first done by Guynn et al. [11] where they modeled a fiber misalignment and by using periodic boundary conditions on the free edges captured the kinking stress. The disadvantage of this is that the angle of the kink band is locked at 0°, which was recently investigated by Romanowicz [12] to have a significant influence of the global response of the composite. The same type of periodic boundary conditions was used again by Gutkin et al. [13] where they continued the analysis into the postkinking regime.

In [14] a 2D finite element scheme was used to model fiber misalignment with free edges in a geometrical and material non-linear analysis to capture the strain localization in a kink band. They investigated different types of fiber misalignments, variable matrix volume fraction and the effect of material non-linearity of the fibers which was found to affect the critical strain only. In [15] and [16] they further developed the model to take 3D effects into consideration by individual discretization of fibers and matrix in a 3D representative volume element.

Alternative formulations of kink band instabilities include the model in [17] based on elastic planar finite deformation analysis.

The aim of this paper is to compare and validate the constitutive model made by Christoffersen and Jensen [6] with a finite element model comparable to [14]. The comparison is made for the kinking stress, the global response, the fiber angle in the kink band and the kink band orientation for different initial fiber misalignments.

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2. Constitutive equations

It was observed in [6] and [14] that the critical stress was only slightly affected by nonlinearity in the fibers. Due to these findings, it is assumed in this paper that only the matrix behaves elastic–plastic, while the fibers remain elastic. It should be emphasized that it does not impose difficulties in the present analysis to include fiber non-linearities. However, since the main purpose of the present analysis is to compare with previous results where fibers were assumed linear elastic, this assumption is also introduced here. The plasticity of the matrix material is described by the J_2-flow theory with isotropic hardening. The time-independent constitutive tensor L_{ijkl} relating the Jaumann rate of Kirchhoff stresses $\hat{\tau}_{ij}$ to strain rate $\dot{\varepsilon}_{ij}$ is

$$\hat{\tau}_{ij} = L_{ijkl}\dot{\varepsilon}_{kl} \tag{1}$$

where the constitutive tensor L_{ijkl} for J_2-flow theory from [18] using a finite strain formulation (total Lagrangian) as in [19] is

$$L_{ijkl} = \frac{E}{1+\nu}\left(\frac{1}{2}(G_{ik}G_{jl}+G_{il}G_{jk}) + \frac{\nu}{1-2\nu}G_{ij}G_{kl} - \beta^*\frac{3}{2}\frac{E/E_t-1}{E/E_t-(1-2\nu)/3}\frac{s_{ij}s_{kl}}{\sigma_e^2}\right) - \frac{1}{2}(G_{ik}\tau_{jl}+G_{jk}\tau_{il}+G_{il}\tau_{jk}+G_{jl}\tau_{ik}) \tag{2}$$

where G_{ij} are the components of the metric tensor of the deformed configuration, E is the Young modulus of elasticity and E_t is the tangent modulus. s_{ij} are the components of the deviatoric stress tensors and is defined by Kirchhoff stresses τ_{ij} as

$$s_{ij} = \tau_{ij} - \frac{1}{3}G_{ij}G^{kl}\tau_{kl} \tag{3}$$

σ_e is the equivalent von Mises stress:

$$\sigma_e = \sqrt{\frac{3}{2}G^{ik}G^{jl}s_{ij}s_{kl}} \tag{4}$$

The relation between Kirchhoff and Cauchy stresses is

$$\sigma_{ij} = \sqrt{\frac{g}{G}}\tau_{ij} \tag{5}$$

where g and G are the determinants of the metric tensor of the undeformed and deformed configuration, respectively. β^* is determined by

$$\beta^* = \begin{cases} 1 & \text{for } \sigma_e = (\sigma_e)_{\max} \text{ and } \dot{\sigma_e} \ge 0 \\ 0 & \text{for } \sigma_e < (\sigma_e)_{\max} \text{ or } \dot{\sigma_e} < 0 \end{cases} \tag{6}$$

The relation between the uniaxial logarithmic strain ε and the uniaxial Cauchy stress σ is described as a Ramberg–Osgood relation for the matrix material by

$$\varepsilon = \frac{\sigma}{E} + \frac{3\sigma^y}{7E}\left(\frac{\sigma}{\sigma^y}\right)^n \tag{7}$$

where σ^y is the yield stress and n is the hardening index. The tangent modulus E_t is determined by differentiation of (7). The actual values for the material parameters shown later are motivated by experimental findings in [14] for in-situ PEEK reinforced by carbon fibers tested uniaxially as well as in shear.

3. The discretized model for fiber composites

The finite element model is built in a comparable scheme to [14]. The commercial finite element code Marc from MSC Software is used for the analysis. The numerical scheme is chosen as an updated Lagrangian formulation. The model is built of alternating fiber and matrix layers with one 8 node bi-quadratic plane strain element per individual fiber and matrix layer. It was observed by Borg [20] that using 1 element per layer compared to 3 gave a deviation on the kink stress by only 3%. The grid in [14] is also made of 1 element per layer. A convergence study is conducted to determine the necessary number of fiber/matrix layers in a representative volume element that captures the kinking and the steady state postkinking stress accurately. An illustration of the model is shown in Fig. 1. The imperfection to simulate a fiber misalignment is imposed as a cosine function in the area marked by the dashed lines in Fig. 1:

$$x_2 = \frac{h}{2}\left(1 - \cos\left(\frac{\pi x_1}{b}\right)\right) \tag{8}$$

where h is determined from the misalignment angle ϕ_0:

$$h = \frac{2b\tan(\phi_0)}{\pi} \tag{9}$$

and b is the width of the imperfection. The angle of the imperfection β determines in combination with b the area where the imperfection from (8) applies. The fibers outside this area are straight. When $b = L_0$ there are no straight fibers and the misalignment is applied to the whole model. This type of misalignment is in this paper referred to as *global* imperfection. When $b < L_0$, the misalignment is referred to as *local* imperfection.

The fiber volume fraction c^f determines the matrix volume fraction c^m by

$$c^f + c^m = 1 \tag{10}$$

which may change during deformation as the strains and material properties in the fibers and matrix differ. The width, W_0, and the length, L_0, are fixed values in the analysis. This leads to the width of each individual fiber being a variable determined by the number of fibers in the model and c_f. The fiber width then controls the mesh of the model while the matrix width also is controlled by the fiber width.

Since the equilibrium path may experience snap-through and snap-back, the numerical technique for incremental solution is chosen as the arc-length method first introduced by Riks [21]. A linear constraint is chosen as described in [22] so that the sub-increment $(\delta\boldsymbol{u}, \delta\boldsymbol{f})$ lies in a hyperplane orthogonal to the current total increment $(\Delta\boldsymbol{u}, \Delta\boldsymbol{f})$ and is expressed by the condition:

$$(\Delta\boldsymbol{u}, \Delta\boldsymbol{f}) \cdot (\delta\boldsymbol{u}, \delta\boldsymbol{f}) = 0 \tag{11}$$

where $(\boldsymbol{u})$ is the displacement vector and $(\boldsymbol{f})$ is the force vector. This load factor increment is calculated as

$$\delta\xi = -\frac{\Delta\boldsymbol{u}^T\delta\boldsymbol{u}_r}{\Delta\boldsymbol{u}^T\Delta\boldsymbol{u}_1} \tag{12}$$

where $\delta\boldsymbol{u}_r$ is the sub-increment residual displacement vector and $\Delta\boldsymbol{u}_1$ is the initial displacement vector in the current increment.

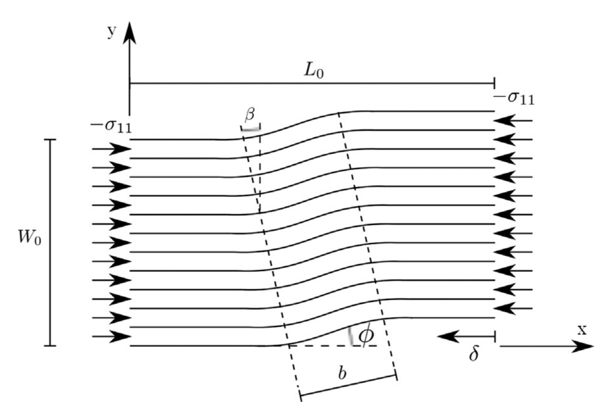

Fig. 1. Numerical setup.