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Bifurcations and route to chaos for flow over an oscillating airfoil

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A B S T R A C T

Using immersed boundary methods, we perform numerical simulations for flow over an elliptical airfoil oscillating at a reduced frequency of 6.283 over a range of Strouhal number with Reynolds number 500. We quantify the response of this nonlinear system through time-histories of aerodynamic lift and thrust force coefficients, their Fourier spectra, phase maps, Poincáre sections, and higher-order spectral analysis. When the oscillation amplitude increases, this nonlinear system undergoes bifurcations, thus eventually achieving chaotic solutions. The aerodynamic response moves quasi-periodically through to chaos. We also relate these behavioral changes in the response of the system to the wake transitions. We also present cross-bispectrum between the lift and thrust coefficients, and auto-trispectrum for the lift coefficient to study cubic coupling among various frequency components. We find the strongest quadratic and cubic coupling between the excitation frequency components. We also observe the cubic coupling among various frequency components responsible for generating new harmonics in the spectra of the lift force at higher Strouhal numbers.

1. Introduction

Since the discovery of a special vortex pattern, known as reverse von Kármán vortex street by Knoller (1909) and Betz (1912) in the wake of oscillating wings to generate thrust, many experimental (von Ellenrieder et al., 2003; Jones and Platzer, 1999; Koochesfahani, 1989; Triantafyllou et al., 2004) and numerical studies (Narasimhan et al., 2006; Asharf et al., 2011; Jones and Platzer, 1997; Yu et al., 2013b) have been presented in literature related to the underlying flow structure. These were mostly devoted to understand the effect of various parameters like the oscillation amplitude ($Y_\circ$), the excitation frequency (f_E), the Reynolds number (Re), and the geometric features of the airfoils on the associated production, shedding and interaction of vortices. It is customary to define the motion of oscillating structures in the form of two important non-dimensional governing parameters known as, reduced frequency ($k = 2\pi f_E c/U_\infty$) and Strouhal number ($St = fA_\circ/U_\infty$), where f_E, c, U_∞, and A denote the excitation/oscillation frequency, chord-length of the airfoil, free-stream velocity, and the maximum amplitude traversed by the airfoil's trailing-edge during its one oscillation, respectively. Lewin and Hariri

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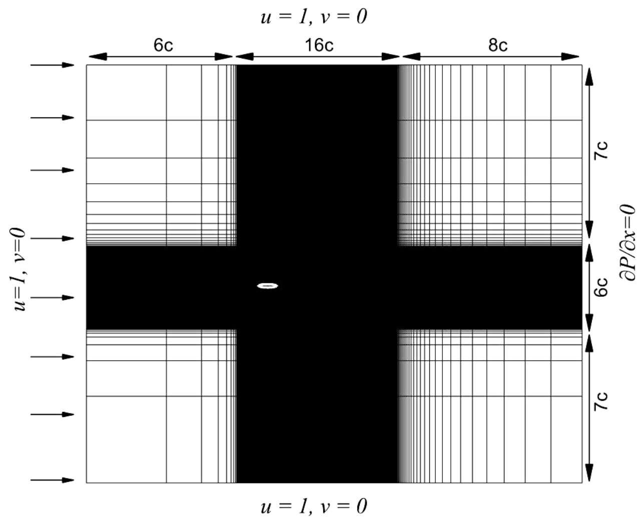

Fig. 1. Schematic of the flow domain and a representation of the boundary conditions.

Table 1
Details of mesh sizes to examine grid-independence.

Scheme	N_X	N_Y	$\bar{C_T}$
$\mathbb{G}_1$	869	568	−0.035
$\mathbb{G}_2$	1079	778	−0.029
$\mathbb{G}_3$	1269	948	−0.029

(2003) investigated the effect of reduced frequency and Strouhal number on the formation of the leading-edge vortex (LEV) and its interaction with the trailing-edge vortex (TEV) in the wake of a plunging airfoil. The geometry of this airfoil was based on a Joukowsky transformation of a circle with a round trailing edge. They observed periodic and symmetric (for $2 \leq k \leq 5$ and $0.25 \leq St \leq 0.48$), asymmetric (for $5 < k < 6$ and $0.30 < St < 0.50$) and aperiodic wake structures (for $k > 6$ and $St > 0.35$) at $Re = 500$. Chaotic flow structures in the wake of a plunging foil in the hovering mode for Re ranging from 48 to 64 were observed by Blondeaux and Guglielmini (2005). They found that the chaotic solution was transitioned from periodicity, quasi-periodicity and phase-locking mechanisms. Young and Lai (2007) proposed the boundaries for the fundamental synchronization known as lock-on between the excitation and wake frequencies for flow over a plunging NACA0012 airfoil at $Re = 2 \times 10^4$. The nature of this boundary was nonsymmetrical about the natural frequency of the system due to the involvement of a sharp trailing-edge with the on-coming flow for $2.0 \leq k \leq 20.0$ (modified according to the present definition) and a low nondimensional amplitude $Y_\circ/c = 0.10$. For a higher frequency ($k = 10.0$) and the lower ranges of the oscillation amplitude ($0.005 \leq Y_\circ \leq 0.05$), Visbal (2009) reported the dynamic stall suppression for a plunging SD7003 airfoil at an angle-of-attack $\alpha_\circ = 4°$. He performed high-fidelity numerical simulations for these flows at $Re = 10^4$ up to 6×10^4. He did not observe any chaotic flow-structure in the wake with three-dimensional simulations. Based on these investigations, Visbal (2009) argued that the chaotic solution was an artifact of two-dimensional (2D) flow-fields that did not exist in 3D. This was confirmed experimentally by Cleaver et al. (2009, 2010, 2011). Ashraf et al. (2012) reported the presence of a chaotic vortical structure for a higher amplitude ($Y_\circ = 1.5c$) and a lower frequency ($k = 2$). Measuring the response of this nonlinear system in terms of the thrust coefficient (C_T), they observed periodic, quasi-periodic and chaotic solutions, respectively by increasing $Y_\circ$ due to the complex interaction between LEVs. Cleaver et al. (2012) performed experiments at $Re = 10^4$ to explore the effect of increasing frequencies and with slightly increasing oscillation amplitudes on the vortical wake, and the resulting impact on its time-averaged lift-coefficient ($\bar{C_L}$) where "¯" is used here to indicate an averaged quantity. They reported the occurrence of significant bifurcations in the response curve of $\bar{C_L}$ with slow variations in the frequency. They also showed the existence of stable branches in these response diagrams which were related to upward and downward deflecting wakes. Solving Navier–Stokes equation numerically using a higher-order spectral difference method, a bifurcated wake was also shown by Yu et al. (2013a) for a root-fixed finite-span rectangular flapping wing at $k = 3.4$, $St = 0.38$ and $Re = 1.2 \times 10^3$. Recently, Calderon et al. (2014) explored the nonexistence of asymmetric wakes for finite-span plunging NACA0012 wings at the high end of frequency range ($k \leq 6.36$) for $Y_\circ = 0.15c$ at $Re = 10^4$. They argued that the tendency of the wake to become asymmetric seemed to be suppressed by the effect of three-dimensionality.

These afore-mentioned studies give the results related to qualitative changes in the flow-field and the resulting impact on the aerodynamic forces due to the variations in the kinematic parameters of oscillating wings. In these flow-fields, vortical