Journal of Fluids and Structures 80 (2018) 390–404

Contents lists available at ScienceDirect

Journal of Fluids and Structures

journal homepage: www.elsevier.com/locate/jfs

A nonlinear model for a free-clamped cylinder subjected to confined axial flow

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ARTICLE INFO

Article history:
Received 16 August 2017
Received in revised form 9 February 2018
Accepted 14 March 2018

Keywords:
Free-clamped cylinder
Nonlinear dynamics
Axial flow
Instability

ABSTRACT

In this paper a full nonlinear model[1] is presented for the dynamics of a cantilevered cylinder, terminated by an ogival free end, and subjected to confined, inverted axial flow. This system is also known as "a free-clamped cylinder in axial flow", since the flow is directed from the free end towards the clamped one. All the fluid-related forces and the gravity-related terms are derived separately to third-order accuracy; the inviscid forces are modelled using an extension of Lighthill's slender-body analysis to the same accuracy, and the viscous forces are obtained semi-empirically. The boundary conditions related to the free end are also derived separately, to first-order accuracy, and added to the model. The final equation of motion is obtained via Hamilton's principle, then discretized and solved numerically using AUTO and MATLAB software. The stability of the system is investigated by means of bifurcation diagrams, time histories, phase-plane and power-spectral-density plots, and the dynamical behaviour is compared to theoretical predictions and experimental observations, from the literature, for systems that have the same parameters. The theory is in good qualitative agreement with the experiments, and also good quantitative agreement in terms of the critical flow velocity of instability.

1. Introduction

Interest in studying the dynamics of cylinders subjected to axial flow began with the development of the Dracone barge in the late 1950s. The Dracone is a long flexible tubular container, which has been designed for the sea-transport of liquids lighter than sea-water, such as oil and fresh water. Perhaps the first specific study of the Dracone, or generally towed cylinders in axial flow, was undertaken by Hawthorne (1961). Païdoussis (1966a) extended and generalized Hawthorne's analysis for cylinders with different boundary conditions, and supported the theory with experiments in Païdoussis (1966b). An error in incorporating the viscous forces into the equation of motion was realized and a corrected form of the equation of motion was derived by Païdoussis (1973); in addition, the theory was further extended to deal with the case of a cluster of cylinders in axial flow. That case has been extensively studied afterwards, e.g. in Chen (1975), Païdoussis et al. (1977) and Païdoussis (1979); see also Païdoussis (2016) for a more detailed review. The dynamics of cantilevered cylinders in axial flow was examined in Païdoussis et al. (2002), Lopes et al. (2002) and Semler et al. (2002) by a nonlinear theory. In that three-part study, the physical dynamics of the system were also discussed, which are quite similar to the problem at hand.

The dynamics of a free-clamped cylinder in axial flow has been studied for the first time by Rinaldi and Païdoussis (2012). In that study, experiments were conducted on a cylinder, fitted with differently shaped end-pieces, in confined axial air-flow. Small-amplitude first-mode, flutter-like oscillations were observed at relatively low flow velocities; and with increasing the

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[1] A model that encompasses nonlinear fluid dynamics as well as nonlinear structural dynamics.

https://doi.org/10.1016/j.jfluidstructs.2018.03.006
0889-9746/

flow velocity, the amplitude of the oscillations diminishes and a static bowing starts to develop. In addition, a simple linear theoretical model was derived in the same study, which predicts only a buckling instability with increasing the flow velocity, and it overestimates the onset of instability with respect to the one observed experimentally.

More recently, some emphasis has been placed on this "inverted" configuration, but for flags and flexible plates, such as in Kim et al. (2013), Ryu et al. (2015), Tang et al. (2015) and Gurugubelli and Jaiman (2015), some studies aimed to harvest energy using piezoelectric inverted flags, e.g. Shoele and Mittal (2016) and Orrego et al. (2017). Those studies concluded that flags exhibit large-amplitude flapping over finite bands of flow speeds. The physical mechanisms leading to such behaviour were explored by Sader et al. (2016a) who mathematically proved that flapping is initiated by a divergence instability, but again, the theory overestimated the critical flow velocity of the divergence instability for slender inverted flags. In another study, Sader et al. (2016b) explored the possibility of the existence of a vortex-lift mechanism, and employed the work of Bollay (1939) and Taylor (1952) who calculated the steady hydrodynamic forces on rectangular blades and cylinders. Sader et al. (2016b) also studied the case of an inverted cylinder with the same parameters as in Rinaldi and Païdoussis (2012), applied a theoretical static analysis, and concluded that the cylinder undergoes a saddle–node bifurcation that leads to another stable static solution at a lower critical flow velocity compared to the theory in Rinaldi and Païdoussis (2012). Most recently, the mechanism of flapping of inverted flags and foils has been further analysed in Goza et al. (2017) and Gurugubelli and Jaiman (2017).

In the present study, a full nonlinear analytical model is derived for a free-clamped cylinder in axial flow taking into account the effects of the confinement of the flow and also the boundary conditions related to the free end of the cylinder. The fluid-related forces considered are the inviscid hydrodynamic forces, the hydrostatic forces and the viscous forces. Moreover, a vortex-lift mechanism is also considered and the associated steady hydrodynamic forces are derived on the basis of the semi-empirical expressions proposed by Taylor (1952). The weakly nonlinear equation of motion is derived in Section 2, which is exact to third-order of magnitude. The equation is solved using the same system parameters as in Rinaldi and Païdoussis (2012), and the nonlinear dynamics of the system is examined in Section 3. In Section 4, the results of the proposed model are compared to the experimental observations reported in Rinaldi and Païdoussis (2012) and to the results of other theoretical models from the literature.

2. Derivation of the equation of motion

The system under study consists of a flexible cantilevered cylinder of diameter D, length L, flexural rigidity EI and mass per unit length m. The cylinder is centrally located in a rigid channel of diameter D_{ch}, as shown in Fig. 1a, and is subjected to an axial flow velocity U, which is directed from the free end towards the clamped one. The system is vertical, so the undeformed axis of the cylinder coincides with the X-axis and is in the direction of the gravity, as shown in Fig. 1b. In addition, the cylinder is generally fitted with an ogival end-piece at the free end of the cylinder.

The following basic assumptions are made for the cylinder and the fluid: (i) the cylinder length-to-diameter ratio is high enough for the Euler–Bernoulli beam theory to apply; (ii) the cylinder centreline is inextensible; (iii) the strains of the cylinder are small, but the deflections can be large; (iv) the motion of the cylinder is assumed to be planar; and (v) the fluid is incompressible with constant mean flow velocity.

Two coordinate systems are used: the Lagrangian (X, Y, Z, t) and the Eulerian (x, y, z, t); the former one is associated with the undeformed state of the cylinder, while the latter is for the deformed state. The displacements of point G on the undeformed cylinder are thus $u = x - X$, $v = y - Y$, and $w = z - Z$, as shown in Fig. 1b. The cylinder centreline motions are assumed to be in the $(X$-$Y)$ plane, hence $Y = 0$ and $z = Z = w = 0$. The curvilinear coordinate along the cylinder, s, can be related to X by $\frac{\partial s}{\partial X} = 1 + \bar{\varepsilon}$, where $\bar{\varepsilon}$ is the axial strain along the centreline with $1 + \bar{\varepsilon}(X) = [(\frac{\partial x}{\partial X})^2 + (\frac{\partial y}{\partial X})^2]^{1/2}$. The cylinder centreline is assumed to be inextensible, hence $\bar{\varepsilon} = 0$, $\frac{\partial s}{\partial X} = 1$ and $(\frac{\partial x}{\partial X})^2 + (\frac{\partial y}{\partial X})^2 = 1$.

The equation of motion is derived via Hamilton's principle,

$$\delta \int_{t_1}^{t_2} \mathcal{L}\,\mathrm{d}t + \int_{t_1}^{t_2} \delta W\,\mathrm{d}t = 0, \tag{1}$$

where $\mathcal{L} = \mathcal{T}_c - \mathcal{V}_c$ is the Lagrangian, $\mathcal{T}_c$ is the kinetic energy of the cylinder, $\mathcal{V}_c$ is its potential energy, and δW is the virtual work acting on the cylinder by the fluid-related forces. The derived equation is correct to third-order of magnitude, $\mathcal{O}(\epsilon^3)$, for $y = v \sim \mathcal{O}(\epsilon)$ and $u \sim \mathcal{O}(\epsilon^2)$. Hence the expression for the virtual work must be correct to $\mathcal{O}(\epsilon^3)$, while the energy expressions to $\mathcal{O}(\epsilon^4)$.

2.1. Kinetic and potential energies of the cylinder

The kinetic and potential energies of the cylinder itself are

$$\mathcal{T}_c = \frac{1}{2}m\int_0^L V_c^2\mathrm{d}X, \quad \mathcal{V}_c = \frac{1}{2}EI\int_0^L \bar{\kappa}^2\mathrm{d}X - mg\int_0^L x\mathrm{d}X, \tag{2}$$

where V_c is the velocity of the cylinder element and can be expressed as $\vec{V}_c = \dot{x}\vec{i} + \dot{y}\vec{j}$, in which $\vec{i}$ and $\vec{j}$ represent axial and lateral directions of the undeformed state of the cylinder, respectively, as shown in Fig. 2. x and y are related to each other