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Stability of helical tubes conveying fluid

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ABSTRACT

We study the linear stability of elastic collapsible tubes conveying fluid, when the equilibrium configuration of the tube is helical. A particular case of such tubes, commonly encountered in applications, is represented by quarter- or semi-circular tubular joints used at pipe's turning points. The stability theory for pipes with non-straight equilibrium configurations, especially for collapsible tubes, allowing dynamical change of the cross-section, has been elusive as it is difficult to accurately develop the dynamic description via traditional methods. We develop a methodology for studying the three-dimensional dynamics of collapsible tubes based on the geometric variational approach. We show that the linear stability theory based on this approach allows for a complete treatment for arbitrary three-dimensional helical configurations of collapsible tubes by reduction to an equation with constant coefficients. We discuss new results on stability loss of straight tubes caused by the cross-sectional area change. Finally, we develop a numerical algorithm for computation of the linear stability using our theory and present the results of numerical studies for both straight and helical tubes.

1. Background of the studies in dynamics of flexible tubes conveying fluid

The dynamics of tubes conveying fluid poses many interesting problems in both applied and fundamental mechanics, in addition to its practical importance for engineering applications. For such systems, an instability appears when the flow rate through the tube exceeds a certain critical value. While this phenomenon has been known for a very long time, the quantitative research in the field started around 1950 (Ashley and Haviland, 1950). Benjamin (1961a, b) was perhaps the first to formulate a quantitative theory for the 2D dynamics of the *initially straight tubes* by considering a linked chain of tubes conveying fluids and using an augmented Hamilton principle of critical action that takes into account the momentum of the jet leaving the tube. A continuum equation for the linear disturbances was then derived as the limit of the discrete system. This linearized equation for the initially straight tubes was further considered by Gregory and Païdoussis (1966a).

These initial developments formed the basis for further stability analysis of this problem for finite, initially straight tubes (Païdoussis, 1970; Païdoussis and Issid, 1974; Païdoussis, 1998; Shima and Mizuguchi, 2001; Doaré and de Langre, 2002; Païdoussis and Li, 1993; Païdoussis, 2004; Akulenko et al., 2013, 2015, 2016). The linear stability theory has shown a reasonable agreement with experimentally observed onset of the instability (Gregory and Païdoussis, 1966b; Païdoussis, 1998; Kuronuma and Sato, 2003; Castillo Flores and Cros, 2009; Cros et al., 2012). Nonlinear deflection models were also considered in Li et al. (1994), Païdoussis (2004), Modarres-Sadeghi and Païdoussis (2009), Ghayesh et al. (2013), and the compressible (acoustic) effects in the flowing fluid in Zhermolenko (2008). Alternatively, a more detailed 3D theory of

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motion was developed in Beauregard et al. (2010) and extended in Rivero-Rodriguez and Perez-Saborid (2015), based on a modification of the Cosserat rod treatment for the description of elastic dynamics of the tube, while keeping the cross-section of the tube constant and orthogonal to the centerline. In particular, Rivero-Rodriguez and Perez-Saborid (2015) analyzes several non-straight configurations, such as tube hanging under the influence of gravity, both from the point of view of linear stability and nonlinear behavior. Unfortunately, this Cosserat-based theory could not easily incorporate the effects of the cross-sectional changes in the dynamics. Some authors have treated the instability from the point of view of the *follower force approach*, which treats the system as an elastic beam, ignoring the fluid motion, with a force that is always tangent to the end of the tube. Such a force models the effect of the jet leaving the nozzle (Bou-Rabee et al., 2002). However, once the length of the tube becomes large, the validity of the follower force approach has been questioned, see Elishakoff (2005) for a lively and thorough discussion. For the history of the development of this problem in the Soviet/Russian literature, we refer the reader to the monograph (Svetlitskii, 1987, still only available in Russian). To briefly touch upon the developments in Russian literature that have been published in parallel with their western counterparts, we refer the reader to the selection of papers (Movchan, 1965; Mukhin, 1965; Ilgamov, 1969; Anni et al., 1970; Vol'mir and Gratch, 1973; Svetlitskii, 1978; Dotsenko, 1979; Chelomey, 1984; Sokolov and Bereznev, 2005; Amenzade and Aliev, 2015).

Because of its importance for practical applications, the theory of curved pipes conveying fluid has been considered in earlier works in some detail. The equations of motion for the theory were derived using the balance of elastic forces from tube's deformation and fluid forces acting on the tube when the fluid is moving along a curved line in space. In the western literature, we shall mention the earlier work (Chen, 1972), followed with more detailed studies (Misra et al., 1988a, b; Dupuis and Rousselet, 1992) which developed the theory suited for both extensible and inextensible tubes and discussed the finite-element method realization of the problem. We shall also mention (Doll and Mote, 1976; Aithal and Gipson, 1990) deriving a variational approach for the planar motions of initially circular tubes, although the effect of curved fluid motion was still introduced as extra forces through the Lagrange–d'Alembert principle. In the Soviet/Russian literature, Svetlitskii (1978) developed the rod-based theory of oscillations and Sokolov and Bereznev (2005) considered an improved treatment of forces acting on the tubes. Most of the work has been geared towards the understanding of the planar cases with in-plane vibrations as the simplest and most practically relevant situations (still, however, leading to quite complex formulas).

In spite of considerable progress and understanding achieved so far, we believe that there is still much room for improvement in the theoretical treatment of the problem. In particular, the derivation of the theory based on the balance of forces is not variational and the approximations of certain terms tend to break down the intrinsic variational structure of the problem. In contrast, the theory of flexible tubes conveying fluid as developed in Gay-Balmaz and Putkaradze (2014, 2015) is truly variational and all the forces acting on the tube and the fluid, as well as the boundary forces are derived automatically from the variational principle. More importantly, it is very difficult (and perhaps impossible) to extend the previous theory to accurately take into account the changes in the cross-sectional area of the tube, also called the collapsible tube case. In fact, we are not aware of any studies on the subject of stability for initially curved collapsible tubes, especially undertaken from a variational point of view.

In many previous works, the effects of cross-sectional changes have been considered through the quasi-static approximation: if $A(s,t)$ is the local cross-section area, and $u(s,t)$ is the local velocity of the fluid, with s being the coordinate along the tube and t the time, then the quasi-static assumption states that $uA =$ const, see, *e.g.*, Li et al. (1994) and Ghayesh et al. (2013). Unfortunately, this simple law is not correct in general and should only be used for steady flows. This problem has been addressed by two of the authors of this paper in Gay-Balmaz and Putkaradze (2014, 2015), where a geometrically exact setting for dealing with a variable cross-section was developed and studied, showing the important effects of the cross-sectional changes on both linear and nonlinear dynamics. The nonlinear theory was derived from a variational principle in a rigorous geometric setting and for general Lagrangians. It can incorporate general boundary conditions and arbitrary deviations from equilibrium in the three-dimensional space. From a mathematical point of view, the Lagrangian description of these systems involves both left-invariant (elastic) and right-invariant (fluid) quantities. The theory derived in Gay-Balmaz and Putkaradze (2014, 2015) further allowed consistent variational approximations of the solutions, both from the point of view of deriving simplified reduced models and developing structure preserving numerical schemes (Gay-Balmaz and Putkaradze, 2016).

In this work, we undertake a detailed study of the fully three dimensional vibrations for the problem when the equilibrium spatial configuration of the centerline for the tube is helical, and the cross-sectional area of the tube is allowed to change. Since a circular arc is a particular case of a helix, the linear stability of a tube with centerline having a circular arc can be considered as a particular case of our studies. The geometric approach underpinning the theory developed in Gay-Balmaz and Putkaradze (2014, 2015) considers the dynamics in the framework of the group of rotations and translations. This, in turn, allows for the complete analysis of the stability of an initially helical tube by reducing it to a system of equations with constant coefficients. To put it in simpler terms, the geometric framework unifies the concept of the stability analysis of the initially helical and straight tubes. Of course, the stability analysis of the helical tubes is much more complicated as compared to the straight ones; nevertheless, a substantial analytic progress can still be achieved in the more complex case of initially helical tube as well, which is precisely the focus of this paper.

2. Mathematical preliminaries and background of the variational method

2.1. Introduction to geometric variational methods

In this Section, we shall outline the background of the method and introduce some useful notations. We will try to make this Section self-consistent so the reader unfamiliar with the variational methods can follow the derivation of Section 3