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Size-dependent post-buckling behaviors of geometrically imperfect microbeams

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ABSTRACT

In this paper, the size-dependent buckling and post-buckling analysis of the imperfect microbeams is presented. By accounting the mid-plane stretching, the nonlinear integral partial-differential equation governing the transverse motion of the imperfect microbeam is derived employing Hamilton's principle. The modified couple stress theory is employed to capture the size effects of microbeams. The exact analytical solutions of the critical buckling load and the static response are acquired for the buckling problem of imperfect microbeams with different boundary conditions. Moreover, the dynamic behavior around the static configurations in the preand postbuckling domain is also investigated. It is found that the buckling of imperfect microbeam occurs through a saddlenode bifurcation, and the responses in postbuckling domain are asymmetrical. The numerical results are depicted to illustrate the influence of the dimensionless scale parameter and the imperfection amplitude on the circuital buckling load, static response and the dynamic properties of the imperfect microbeams.

1. Introduction

Micro-structures are playing an important role in the design of micro- and nano-electromechanical systems (MEMS and NEMS) such as the micro-gripper [1], microswitch [2–4], microactuators [5,6] and microsensors [7]. Those structure components used in miniaturized devices are typically on the order of micrometer or even nanometer. The size effects, which arise from the existence of quantum effects and high surface to volume ratio in nano- and micro-structure, have been observed in the experiments and atomistic simulations [8,9]. It is well known that the size-dependent behavior cannot be predicted by the classical continuum theories, due to the fact that the classical continuum theories do not include material length scales. Therefore, various scale-dependent continuum theories such as strain gradient theories [10,11], the couple stress theories [12–14], nonlocal elasticity [15] and surface elasticity theory [16] are applied to capture the size effect in the small scale structures. Among those size-dependent theories, the couple stress theory employ two additional material length-scale parameters other than the classical Lame constants in the constitutive equa- tion to take into account the size-dependent behavior. Recently, the classical couple stress theory has been modified by Yang et al. [17]. The modified couple stress theory (MCST) needs only one additional material length-scale parameter to capture the size-dependency. This feature made the MCST was widespread applied in the micro- or nano-scale investigations. For example, Park et al. [18] investigated the bending of Euler-Bernoulli microbeams by employing the MCST. The results showed that the size effect predicted by MCST had good agreement with the results of the experiment. Kong et al. [19] and Ma et al. [20] utilized the MCST based Euler-Bernoulli and Timoshenko model to study the free vibration of microbeams, respectively. They showed that the size effect will be significant when the thickness of beam is comparable to material length scale parameter, but it will diminish with the increase of the thickness. Akgoz et al. [21] presented buckling analysis of micro-scaled Euler-Bernoulli beams by using the strain gradient and modified couple stress theories. The size dependent buckling of functionally graded microbeams was presented by Nateghi et al. [22] using three different beam theories, including the classic Euler-Bernoulli beam theory, the first and third order shear deformation beam theories. Furthermore, Şimşek et al. [23] developed a unified MCST based higher order beam theory for the static bending and free vibration analysis of functionally graded (FG) microbeams.

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In the past few years, many researchers have focused on the nonlinear problems of microbeams, in which the nonlinear mathematical model was employed in investigations. For instance, Xia et al. [24] developed a nonlinear Euler-Bernoulli beam model for the static bending, postbuckling and vibration problems of microbeams by applying MCST. On the basis of the MCST, a nonlinear Timoshenko beam model for microbeams was presented by Asghari et al. [25]. The authors also presented an exact solution of buckling problem of Euler-Bernoulli microbeam and indicated that many internal resonances might exist around the buckled configurations [26].

Ghayesh et al. [27] investigated nonlinear primary resonant responses of microbeam based on the MCST. The results were based on single mode approximation. The authors expanded their work to coupled vibration of microbeams [28], by taking into account both the transverse and longitudinal motions of beams. The size-dependent dynamical behavior of microbeams with modal interactions were analyzed by Ghayesh and coworkers [29,30] applying the MCST.

In view of various environmental and manufacturing factors, the geometric imperfections of beam-like structure are common in practice. Fang et al. [31] studied the static deformation of imperfect beams in pre- and postbuckling domain employing analytical and experimental methods. Lacarbonara1 [32] investigated the thermal buckling and the postbuckling vibration of clamped imperfect beams. The analytical solutions of the critical thermal buckling load and the static responses of imperfect beams are presented. Emam [33] presented the static and dynamic response of the clamped composite beams with geometrically imperfection. Results showed that the initial imperfection has significant effect on the static and dynamic behaviors of composite beams. The nonlinear free vibration and buckling behavior of imperfect functionally graded beams resting on nonlinear elastic foundation with simply supported and clamped boundary conditions are examined by Yaghoobi et al. [34]. Recently, the static and dynamic behaviors of imperfect microbeams has drawn great interest of researchers. For instance, Ruzziconi et al. [35] reported an experimental investigation on a MEMS device constituted of a clamped polysilicon microbeam with geometric imperfections. The 1/3 s symmetric superharmonic resonance and the 1/3 s antisymmetric one have been observed experimentally. Dehrouyeh et al. [36] investigated the free vibration of FG imperfect microbeam in which the material properties vary according to a power-law in thickness direction. The nonlinear dynamics responses of geometrically imperfect homogeneous microbeams were performed by Farokhi and Ghayesh [37–40] based on the MCST. In their reports, the Galerkin method was used to reduce the nonlinear partial differential equations to a set of nonlinear ordinary differential equations. Then, the high-dimensional nonlinear ordinary differential system was solved numerically. More recently, Dehrouyeh et al. [41] developed a size-dependent model based on the MCST to examine nonlinear thermal stability behavior of a geometrically imperfect FG Euler-Bernoulli microbeam with clamped boundary condition.

The previous works on the buckling analysis of imperfect microbeams are limited to the critical buckling load and the static response with clamped boundary condition. To the authors' best knowledge, there is no reported work in the literature which investigated the postbuckling vibrations that take place in the vicinity of the static deflection of the buckled imperfect microbeams. This article aims to present an analytical method to investigate the size-dependent static response of imperfect microbeams in pre- and postbuckling domain with different boundary conditions. Moreover, the free vibrations in the postbuckling domain of imperfect microbeams are also examined.

The rest parts of this paper are organized as follows. In Section 2, the size-dependent mathematical model of the imperfect

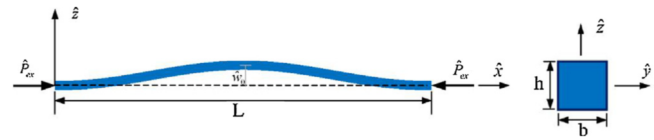

Fig. 1. Geometry of the imperfect microbeam.

microbeams is formulated using Hamilton's principle on the basis of the modified couple stress theory. The Von Karman nonlinear strain is employed to consider the geometric nonlinearity. In Section 3, the closed form solutions of the critical buckling load and the static response of the imperfect micrsbeams with pinned and clamped boundary conditions are acquired by solving the reduced governing equation in the static form analytically. Then, the postbuckling vibrations around the static deflections of the buckled imperfect microbeams are analyzed in Section 4. In Section 5, numerical results are carried out to demonstrate the influence of size effect, initial imperfection on the static and dynamic behaviors of imperfect microbeams with different boundary conditions. Finally, a summary and conclusions are given in Section 6.

2. Mathematical model of microbeam based on modified couple stress theory

A microbeam with initial imperfection is depicted in a rectangular coordinate system as shown in Fig. 1. The length of the microbeam is L,the width is b and the thickness is h. The microbeam is subjected to an axial load of magnitude P_{ex}. w_0 denotes the initial geometric imperfection in $\hat{z}$ direction. It is assumed that the cross-section of the imperfect microbeam is constant along the entire length and there is no warping in the system [40].

According to the modified couple stress theory, the virtual strain energy δU can be written as

$$\delta U = \int_{\Omega} (\sigma_{ij}\delta\varepsilon_{ij} + m_{ij}\delta\chi_{ij})\mathrm{d}V \tag{1}$$

in which,

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + \lambda\delta_{ij}\varepsilon_{kk}, \quad m_{ij} = 2l^2\mu\chi_{ij}, \tag{2a}$$

$$\varepsilon_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}+\frac{\partial u_m}{\partial x_i}\frac{\partial u_m}{\partial x_j}\right), \tag{2b}$$

$$\chi_{ij}=\frac{1}{2}\left(\frac{\partial \theta_i}{\partial x_j}+\frac{\partial \theta_j}{\partial x_i}\right), \quad \theta_i=\frac{1}{2}\in_{ijk}\frac{\partial u_k}{\partial x_j}, \tag{2c}$$

where $i,j,k = 1,2,3$ and summation on repeated subscripts is implied. m_{ij} represent the components of the deviatoric part of the couple stress tensor and χ_{ij} denote the components of the symmetric curvature tensor. Besides, $\mu = \frac{E}{2(1+\nu)}$ and $\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}$ are the Lameí constants in the classical contin- uum theories.

Based on the Euler beam theory, the displacement field is defined as

$$u_1(\hat{x},\hat{z},\hat{t}) = \hat{u}(\hat{x},\hat{t})\text{-}\hat{z}\frac{\partial \hat{w}(\hat{x},\hat{t})}{\partial x}, \tag{3}$$

$$u_2(\hat{x},\hat{z},\hat{t}) = 0, \quad u_3(\hat{x},\hat{z},\hat{t}) = \hat{w}(\hat{x}) + \hat{w}_0(\hat{x}).$$

Then, the nonzero part of strain tensor can be obtained as [40]:

$$\varepsilon_{\hat{x}\hat{x}} = \frac{\partial \hat{u}}{\partial \hat{x}} - \hat{z}\frac{\partial^2 \hat{w}}{\partial \hat{x}^2}+\frac{1}{2}(\frac{\partial \hat{w}}{\partial \hat{x}})^2 + \frac{\partial \hat{w}}{\partial \hat{x}}\frac{\partial \hat{w}_0}{\partial \hat{x}}, \quad \chi_{\hat{x}\hat{y}} = -\frac{1}{2}\frac{\partial^2 \hat{w}}{\partial \hat{x}^2}. \tag{4}$$