Mechanics Research Communications 77 (2016) 50–59

Contents lists available at ScienceDirect

Mechanics Research Communications

journal homepage: www.elsevier.com/locate/mechrescom

Layerwise theory in modeling of magnetorheological laminated beams and identification of magnetorheological fluid

Jalil Naji[a,*], Abolghassem Zabihollah[a], Mehdi Behzad[b]

[a] School of Science and Engineering, Sharif University of Technology, International Campus, Kish, Iran
[b] School of Mechanical Engineering, Sharif University of Technology, Tehran, Iran

ARTICLE INFO

Article history:
Received 13 February 2016
Received in revised form 12 May 2016
Accepted 2 September 2016
Available online 5 September 2016

Keywords:
MR fluid
Laminated beam
Layerwise
Complex shear modulus
Vibration control

ABSTRACT

In recent years, structures integrated with magnetorheological (MR) fluid have been considered for their tunable dynamic characteristics. Shear modulus of MR layer in composite structure is dramatically lower than the elastic layers, leading to high shear deformation inside the MR layer, thus classical theories are not accurate enough to predict the dynamic behavior of such structures. In present study a layerwise displacement theory has been utilized to predict a more accurate deformation for MR-composite beam and equation of motions derived using finite element model (FEM). ASTM E756-98 was employed to evaluate the complex shear modulus of MR fluid. By experimental test a practical formulation for complex shear modulus of MR fluid was extracted. In this formulation shear modulus of MR fluid was presented as function of magnetic flux density and also driving frequency. Based on switched-stiffness vibration control concept a control algorithm was designed and effect of this control strategy on force and free vibration was examined.

1. Introduction

Laminated composite structures have been widely used for many civil, mechanical and aerospace applications for their excellent dynamic performance. Recently, in order to improve the vibration and dynamic characteristics of such structures, smart materials including Magnetorheological (MR) fluids have been integrated to the composite structures. By integrating MR fluids in laminated composite structures, an adaptive structural system with tunable dynamic characteristics can be achieved. The dynamic characteristics of MR adaptive structures such as stiffness, damping, resonance frequencies and amplitude can be tuned by changing external magnetic field.

Gandhi et al. [1,2] had been done first investigations on dynamic characteristics of sandwich beam with Electrorheological (ER) material. Yalcintas et al. [3,4] have been investigated modeling of MR/ER adaptive beam. Sapinski and Snamina [5,6] studied the vibration control capabilities of three-layered cantilever beam with MR fluid. Rajamohan et al. [7,8] utilized the energy method to investigate the dynamic properties of a three layer MR beam. Most recently, Allahverdizadeh et al. [9,10] investigated the vibration analysis of adaptive sandwich beams with ER fluid core using Timoshenko beam theory.

Layerwise displacement fields provides a much more cinematically correct representation of the moderate to severe cross-sectional warping associated with the deformation of thick laminates [11]. Ferreira et al. [12] used layerwise deformation theory to obtain the equation of motions of composite plate. Moreira et al. [13] proposed a layerwise model to simulate the dynamic response of sandwich beam with thin and soft cores like viscoelastic materials. Moreira et al. [14] employed a generalized layerwise formulation to develop a facet-shell finite element able to simulate multiple viscoelastic layer treatments or multiple soft core sandwich plates. Pan and Heyliger [15] derived an analytical solution for vibration of three-dimensional, anisotropic, magneto-electro-elastic, and multilayered rectangular plates. Heyliger and Pan [16] also investigated static behavior of laminates with coupled elastic, electric, and magnetic by the use of a discrete-layer approximate model.Accurate prediction of dynamic response of composite beams with MR layer highly depends on the accurate evaluation of MR materials characteristics, particularly, complex shear modulus and loss factor. Choi et al. [17] investigated elastic modulus and loss factor of the composite beam and those of the ER fluid alone by employing standard test ASTM756-83. Mahjoob et al. [18] exper-

* Corresponding author.
E-mail address: jalil_naji@yahoo.com (J. Naji).

http://dx.doi.org/10.1016/j.mechrescom.2016.09.003

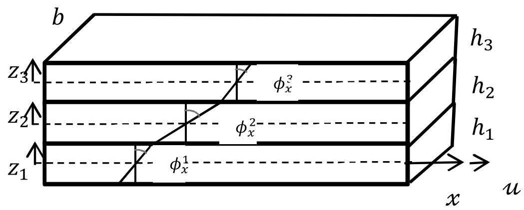

Fig. 1. Layerwise-FSDT model of three-layer beam.

imentally and theoretically studied composite cantilever beams filled with ER material. Leng et al. [19] used ASTM E756-83 standard to experimentally investigate the response characteristics of a beam integrated with ER fluid. Ginder et al. [20] applied numerically and analytically techniques to model the MR fluid and compare the model to magnetorheological experiment. Proposed storage modulus was varying linearly to the magnetic field strength, but they did not present any loss modulus information. Sun et al. [21] by an oscillatory rheometry techniques, developed a relationship between the magnetic field and the complex shear modulus of MR fluid. Eshaghi et al. [22] studied characterization of MR fluids in the pre-yield region and proposed a model for characterizing the complex shear modulus of the MR fluids as a function of both the magnetic flux density and the excitation frequency.

In the present study, dynamic characterization of laminated composite beam structures integrated with MR layer has been studied using layerwise displacement theory. This theory predicted vibration behavior MR-lamented beam more accurate and efficient. A distinctive and innovative formulation for shear modulus of MR fluid was presented. ASTM E756-98 was hired to evaluate the complex shear modulus of MR fluid. Finite Element Model (FEM) has been derived based on the layerwise theory from governing equation of motion. A controller was designed to suppress vibration of MR-beam. Designated controller was employed in force and free vibration of MR-beam.

2. Mathematical modeling of laminated beam with MR fluid layer

Most of the existing works on vibration analysis of laminated beam integrated with MR fluid layer have been developed based on ESL theories, in which the continuous displacement, and thus discontinuous stresses, through the thickness of the laminate are assumed. However, the modulus of MR as middle layer of sandwich beam is usually 10^3 times lower than the face material modulus, requiring a more accurate theory to predict the displacement and stresses through the thickness of the laminate with MR fluid layer.

Reddy's layerwise displacement theory assumes a piecewise displacement through the laminate thickness. However, according to the level of accuracy required for the problem, the displacement through the thickness of each layer may be assumed based on various composite theories including Classical Laminate Plate Theory (CLPT) and First-order Shear Deformation Theory (FSDT) theories.

In the present work, the layerwise displacement theory has been used to predict the dynamic response of the structure. Due to huge difference between the modulus of MR layer and the face material, the displacement of each layer is computed using FSDT. Therefore, the present model is called as layerwise-FSDT.

In proposed model three dependent variables, $(\mathrm{u}_0, \mathrm{w}_0, \phi_{\mathrm{x}}^{(\mathrm{k})})$, are considered. where$u_0$ and w_0 are the translations of the mid-plane of lower layer, $\phi_x^{(k)}$ the rotations of the normal to the mid-plane about the y-axis (Fig. 1).

The displacement field in layers can be defined as:

$$u^{(k)}(x,z) = u_0(x) + \frac{h_1}{2}\phi_x^{(1)}(x) + \sum_{j=2}^{k-1} h_j \phi_x^{(j)}(x) + \frac{h_k}{2}\phi_x^{(k)}(x) + z^{(k)}\phi_x^{(k)}(x) \tag{1}$$

$$w^{(k)}(x) = w_0(x) \tag{2}$$

where h_k are the$k$th layer thickness and $z^{(k)} \in \left[-h_k/2, h_k/2\right]$ are the kth layer z-coordinates.

2.1. Finite element formulation

Considering Hamilton's variational principle as:

$$\delta \int_{t_1}^{t_2} \left(\Pi^K - \Pi^P + W\right) dt = 0 \tag{3}$$

In which Π^{K} and Π^{P} are the kinetic and strain energies of the system respectively, and W is the virtual work. Substituting the displacements into Hamilton's principle, the virtual strain energy is:

$$\delta \Pi^P = \int \sum_{K=1}^{3} \int_{\frac{-h_k}{2}}^{\frac{h_k}{2}} \left(\sigma_{xx}^{(k)} \delta\varepsilon_{xx}^{(k)} + \tau_{xz}^{(k)} \delta\gamma_{xz}^{(k)}\right) dx dz \tag{4}$$

the virtual kinetic energy:

$$\delta \Pi^K = \int \sum_{K=1}^{3} \int_{\frac{-h_k}{2}}^{\frac{h_k}{2}} \rho^{(k)} \left(\dot{u}^{(k)}\, \delta\dot{u}^{(k)} + \dot{w}^{(k)}\, \delta\dot{w}^{(k)}\right) dx dz \tag{5}$$

and virtual work for a transverse distributed load, q on the face layers of the sandwich beam:

$$\delta W = \int q\, \delta w\, dx \tag{6}$$

One may note that in the finite element modeling based on FSDT, only the first derivatives of the dependent variables are required, therefore, Lagrange interpolation functions can be used for approximation. According the layerwise-FSDT, the displacements for the laminated composite beam is given as:

$$u(x,t) = \sum_{j=1}^{n} u_j(t) \quad \psi_j^e(x) \tag{7-1}$$

$$w_0(x,t) = \sum_{j=1}^{n} w_j(t) \quad \psi_j^e(x) \tag{7-2}$$

$$\phi_x^{(k)}(x,t) = \sum_{j=1}^{n} \phi_j^{(k)}(t) \quad \psi_j^e(x) \tag{7-3}$$

where $\psi_j^e$are lagrange interpolation functions through the thickness of each layer. The variables $(u, w_0, \phi_x^{(k)})$ represent the axial, lateral displacement and rotation of the k^{th} layer, respectively.

2.2. Equations of motion

By adopting lagrange interpolation and evaluating the Hamilton's variational principle, equations of motion of a three-layer