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Research paper

Line-symmetric motion generators[☆]

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A R T I C L E I N F O

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A B S T R A C T

When a rigid body is axially reflected through a moving line, its image undergoes a so-called *line-symmetricmotion*. The space comprising all possible line-symmetric motions that share a common initial line is a four-dimensional submanifold, denoted M_4, in the special Euclidean group SE(3). Recently, we showed that M_4 may be used to characterize motions of a line-symmetric body that are free of self-spin and sliding, thus lending itself to applications such as remote center of motion devices for minimal invasive surgery and haptic interfaces. Aiming at designing robot mechanisms for these applications, we present in this paper a systematic enumeration of *line-symmetric motion generators* (LSMGs), i.e., robot mechanisms that generate the line-symmetric motion manifold M_4, following a procedure based on symmetric space theory. LSMGs present a ubiquitous line symmetry of their joint axes, thus offering a new understanding of the line-symmetric motions.

1. Introduction

When a rigid body is axially reflected through a moving line, its image undergoes a so-called line-symmetric motion. In other words, a line-symmetric motion is a one-parameter motion generated by half-turns of the reference body about a line ℓ undergoing a one-parameter motion [1], as illustrated in Fig. 1. A line-symmetric motion is uniquely determined by the *basic surface* [2, Ch. 9.7] swept out by $\ell(\mu)$, $\mu \in \mathbb{R}^+$. Historically, line-symmetric motions were probably first studied by Krames [3] and are known as a rich source of special motions, such as the vertical Darboux motion, the Bricard motion, and the Bennett motion, and were extensively studied by using both basic surfaces and axodes [1,2]. Line-symmetric motions found applications in characterizing self-motions of Griffis–Duffy type parallel manipulators [4] and also point-symmetric hexapods [5].

Other studies on line-symmetric motions focus on the structure and properties of the space of all line-symmetric motions that share a common initial line ℓ_0 [1,6,7]. In particular, we recently proved that this space, henceforth denoted M_4, is a four-dimensional submanifold of the rigid displacement group (or *special Euclidean group*) SE(3), and is a *symmetric subspace* [6]. More precisely, as a symmetric space, SE(3) is a differentiable manifold that can be isometrically point-reflected onto itself over any point on the manifold [8]. The point-reflection map or *inversion symmetry* at a point $\mathbf{g} \in SE(3)$, denoted $s_{\mathbf{g}}$, is defined as $s_{\mathbf{g}}(\mathbf{h}) = \mathbf{g}\mathbf{h}^{-1}\mathbf{g}$, $\forall \mathbf{h} \in SE(3)$. A symmetric subspace is a submanifold that is closed under inversion symmetry. As we have shown in [6] and shall recall in the next section, the fact that M_4 is a symmetric subspace gives rise to many important properties that will be useful for the particular application studied in this paper.

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Nomenclature

ℓ, ℓ_0, $\ell(\mu)$, ...	lines represented by unit dual vectors
LSMG	line-symmetric motion generator
ICPM	interconnected parallel mechanism
SE(3)	special Euclidean group
$\mathfrak{se}(3)$	Lie algebra of SE(3)
$\mathfrak{se}(3)^*$	dual space of $\mathfrak{se}(3)$ (wrench space)
$\mathbf{g}, \mathbf{h}, \ldots$	elements of SE(3)
$\boldsymbol{\xi}, \boldsymbol{\eta}, \ldots$	twist vectors in $\mathfrak{se}(3)$
$\boldsymbol{\zeta}, \boldsymbol{\zeta}_i, \ldots$	wrench vectors in $\mathfrak{se}(3)^*$
$[\boldsymbol{\xi}_1, \boldsymbol{\xi}_2]$	commutator of $\boldsymbol{\xi}_1, \boldsymbol{\xi}_2 \in \mathfrak{se}(3)$
$s_{\mathbf{g}}$: SE(3) → SE(3)	inversion symmetry map associated with a point $\mathbf{g} \in$ SE(3)
$\mathbb{R}\mathrm{P}^n$	n-dimensional real projective space
Q_s	Study quadric
$[a_0: a_1: a_2: a_3: b_0: b_1: b_2: b_3]$	Study parameters
M_4	line-symmetric motion manifold
$\mathfrak{m}_4$	Lie-triple subsystem (LTS) of M_4
$\mathfrak{h}_4$	commutator algebra of $\mathfrak{m}_4$
$\mathrm{T}_{\mathbf{g}}\mathrm{M}_4$	(right-trivialized) tangent space of M_4 at $\mathbf{g}$
1, **i, j, k**	basis elements of the quaternion algebra
ε	dual number: $\varepsilon^2 = 0$
Ad($\mathbf{g}$)	adjoint transformation by $\mathbf{g} \in$ SE(3)
$e^{\boldsymbol{\xi}}$	quaternion exponential of $\boldsymbol{\xi} \in \mathfrak{se}(3)$
$\mathcal{L}$	directed line space
$(\boldsymbol{\xi}^+, \boldsymbol{\xi}^-)$	symmetric twist pair (SP) of $\mathfrak{m}_4$
$(\boldsymbol{\xi}_1^+, \ldots, \boldsymbol{\xi}_4^+, \boldsymbol{\xi}_4^-, \ldots, \boldsymbol{\xi}_1^-)$	symmetric twist chain (SC) of $\mathfrak{m}_4$
$\mathcal{M}^-, \mathcal{M}_i^-, \ldots$	distal half of a $\mathfrak{m}_4$-SC
$\mathcal{M}^+, \mathcal{M}_i^+, \ldots$	proximal half of a $\mathfrak{m}_4$-SC
$S^-, S_i^-, \ldots$	twist subspace of $\mathcal{M}^-, \mathcal{M}_i^-, \ldots$
$(S)^\perp$	reciprocal wrench subspace of a twist subspace S

In this paper, we focus on the mechanism synthesis problem, namely, on the synthesis of 4-DoF mechanisms capable of generating arbitrary line-symmetric motions, or in other words, mechanisms whose motion manifolds are open submanifolds of M_4. As far as we are aware, no such mechanisms exist in the literature. Therefore, this paper probably introduces for the first time a class of *line-symmetric motion generators* (LSMGs). As we have demonstrated in Fig. 1 and shall elaborate in Section 2.3, LSMGs can move a line-symmetric object from an initial to a goal configuration by screwing along the common perpendicular of the initial and the goal line, without incurring self spin or sliding about and along its own axis. This kind of motions has apparent applications in robotic motion planning and design for *line-symmetric manipulation* where only the shape of the line-symmetric object is of concern.

The paper is organized as follows. Section 2 gives a brief review of the motion manifold M_4 and discusses its application in characterizing motion of objects with line symmetry. Section 3 proposes a special class of redundant kinematic chains, called symmetric chains, that may generate M_4 under a symmetric movement condition. Section 4 implements the symmetric chains in the synthesis of a class of LSMGs with both in-parallel kinematic structure and interconnecting joints, which we refer to as interconnected parallel mechanisms. Finally, we conclude the paper with a discussion about possible followup work.

2. Line-symmetric motion manifold

2.1. Dual quaternion representation

Following the notation in [1], an element $\mathbf{g}$ of the special Euclidean group SE(3) is mapped into the Study quadric Q_s under dual quaternion representation:

$$Q_s := \left\{ \mathbf{g} = (a_0 + a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}) + \varepsilon(b_0 + b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}) \in \mathbb{R}\mathrm{P}^7 \;\middle|\; \sum_{i=0}^{3} a_i b_i = 0 \right\} \tag{1}$$

where **i, j, k** denote the quaternion units and ε denotes the dual number ($\varepsilon^2 = 0$). For elements $\mathbf{g}$ of SE(3), $\sum_{i=0}^{3} a_i^2 \neq 0$. We also identify elements of the Lie algebra $\mathfrak{se}(3)$ of SE(3) with dual quaternion vectors:

$$\mathfrak{se}(3) := \{(a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}) + \varepsilon(b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}) \mid a_i, b_i \in \mathbb{R}, i = 1, 2, 3\} \tag{2}$$