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Time-dependent reliability analysis for function generation mechanisms with random joint clearances

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ABSTRACT

Time-dependent reliability method for mechanisms predicts the probability of satisfying the motion requirement in a predefined period of time. The current reliability methods do not consider the random clearances in mechanism joints. This work extends the current methods into function generation mechanisms on which the effect of random joint clearances is significant. The motion output is approximated in the first order with respect to random dimension variables and in a higher order with respect to random joint clearances by the Hybrid Dimension Reduction Method. This treatment achieves an optimal balance between accuracy and efficiency. Then an envelope method is used to calculate the time-dependent reliability. The method is demonstrated by the analysis of three four-bar function generation mechanisms.

1. Introduction

Appropriate joint clearances are chosen for ensuring that mechanisms work properly. On the other hand, they may be somewhat uncontrollable due to manufacturing imprecision and wearing [1]. They are in fact stochastic [2], and the uncertainty in clearances can be propagated to the motion output, thereby affecting adversely the kinematic and dynamic performances of mechanisms [1–5]. The effects might be the motion accuracy loss, unreliability, and reduced service life. They become more severe on high-speed and micro-mechanical systems, such as those in aerospace applications, intelligent robots, and numerically controlled machine tools.

Studies on joint clearances include the investigations on their effects on mechanisms' performance and dynamic characteristics [6]. For the latter, the effects of revolute joint clearances on dynamic characteristics are modeled with three major strategies – the massless link approach, the spring–damper approach, and the momentum exchange approach [7]. Among the three approaches, the third approach is more realistic and is widely employed to study the mechanism dynamic with joint clearances. For examples, Erkaya [8] presented a modeling and optimization approach to reduce the undesired effects of joint clearances on a walking mechanism. Another study of Erkaya [9] established a contact model in a revolute joint with clearance by using the nonlinear spring–damper characteristic and then investigated the kinematic and dynamic characteristics of the welding robot manipulator with joint clearance. Flores et al. [10–13] investigated the effects of joint clearances on kinematics and dynamics of planar and spatial mechanisms with rigid and elastic links. Another study of Flores [14] proposed a general and comprehensive approach to automatically adjust the time step with variable time-step integration algorithms, in the vicinity of contact of multi-body systems. Varedi et al. [15] proposed an optimization method to alleviate the undesirable effect of joint clearance. Zhang et al. [16] established a simulation model of joint clearance with the Hertzian normal contact force model and a Coulomb-type friction force model, and then established a polynomial function Kriging meta-model for optimizing the performance of a mechanical system with the revolute joint clearance. Zhang et al. [17]

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Nomenclature

C	Revolute joint
e	Distance between centers of a bearing and a journal
g	Motion error function
$\mathbf{L}$	Vector of random dimension variables
L	A component of $\mathbf{L}$
m	Size of $\mathbf{L}$
$p_f(\theta)$	Point probability of failure at θ
$p_f(\theta_0, \theta_e)$	Interval probability of failure on $[\theta_0, \theta_e]$
p	Number of expansion points
q	Number of random clearance variables
$R(\theta)$	Point reliability at θ
$R(\theta_0, \theta_e)$	Interval reliability on $[\theta_0, \theta_e]$
r	Rank of covariance matrix $\mathbf{\Sigma}$
r_c	Radius of the clearance circle
$\mathbf{S}$	Random variables
$\mathbf{X}$	Vector of the x-coordinates of random clearance variables
X	Component of $\mathbf{X}$
$\mathbf{Y}$	Vector of the $\mathbf{y}$-coordinates of random clearance variables
Y	Component of $\mathbf{Y}$
ε	Allowable motion error
θ	Input angle
μ_g	Mean of the motion error
μ_L	Mean of the dimension variable L
$\boldsymbol{\mu}_z$	Vector of the means of the motion error at expansion points
$\mathbf{\Sigma}$	Covariance matrix of the motion errors at expansion points
σ_g	Standard deviation of the motion error
σ_L	Standard deviation of the dimension variable L
Φ	Cumulative distribution function of a standard normal distribution
ψ	Actual motion output
ψ_d	Desired motion output

established the dynamic equations for a 3-RRR parallel mechanism by using Newton–Euler equations with Lankarani–Nikravesh contact force model and improved Coulomb friction force model, and then investigated the dynamic performances of the 3-dof mechanism with multi-clearance joints.

For kinematic characteristics, the focus is the quantification of the effect of joint clearances on the ability of achieving desired positions or orientations precisely [6]. There are two types of methods in this area. The first type includes deterministic methods, and they are used to specify the mechanism motion error that results from joint clearances without considering the randomness in the joint clearances. In the deterministic approaches, many researchers used a massless virtual link to model the joint clearance and investigated the motion accuracy for the planar mechanisms [1,18,19]. In the error analysis of the spatial mechanisms and manipulators, the virtual work method [6,20], the screw theory method [21–23], and the interval method [24] have been proposed to study the effects of joint clearance on position and orientation deviation of the manipulators. Based on the theory of envelope, Chen [25] presented a geometric method to uniformly construct the indeterminate influences of the input uncertainties and the joint clearance on the pose (position and orientation) deviation of the manipulators. The other type contains probabilistic methods [2,26–31], which rely on probability and statistics for creating stochastic models of joint clearances and the uncertainty propagation. In general, the probability density function (PDF) is used to describe the random behavior of a joint clearance variable in a clearance circle [2]. The uniform distribution and normal distribution are commonly used for the probabilistic model of the joint clearance [2,26]. Although the stochastic approach does not explore the contact kinematic models of the pairing elements of a joint, it is more desirable for important applications where reliability is of great interest.

A reliability method is the major method among the probabilistic methods. The mechanism reliability is the probability of the output member's position and/or orientation falling within a specified range from the desired position and/or orientation [32]. Higher kinematic reliability means a higher chance to achieve the required motion. It is the reason that reliability methods have been extensively applied in mechanism analysis and synthesis [26–39].

There are two types of kinematic reliability, including point kinematic reliability and time-dependent (interval) kinematic reliability [38]. The former reliability is defined at a specific instance of time and can provide instantaneous information at a specific point in the motion interval of a mechanism. Most of the methods in the literature of mechanism reliability analysis and synthesis are for point kinematic reliability, which is usually calculated by the First Order Second Moment (FOSM) method and Monte Carlo Simulation (MCS) [26,28,30–36].