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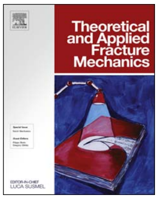

Scrutinized and revised stress intensity factor formulas for double cleavage drilled compression specimens

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ABSTRACT

First, we critically review the stress intensity factor formulas for double cleavage drilled compression (DCDC) specimens, and prove the highly acclaimed analytical formulas for the DCDC were erroneously formulated in Plaisted et al. (2006), we detail the reasons for the errors in several integrals with a short-crack model, and in expressions for bending moment and moment of inertia with a long-crack mode, extending the brief argument in Wang et al. (2016). Then, revised stress intensity factor formulas for DCDC derived by curve-fitting the results from finite element analysis are given, which are applicable to wide-ranging specimen geometries beyond that prescribed in He et al. (1995). Finally, it is shown that even with a load-controlled loading, crack growth stability for DCDC is guaranteed for the whole fracturing process in the experiment, excluding only the DCDC with very small crack length, this is a unique and useful feature pertaining to the DCDC for testing brittle materials.

1. Introduction

The stress intensity factor (SIF) for the specimen entitled DCDC (double cleavage drilled compression) shown in Fig. 1 is critically reviewed and scrutinized. During the last four decades, DCDC has become increasingly popular for fracture test and experiment of quasi-brittle materials [1–19]. Ever since its first use for a class of glasses in the 1970's [1], many researchers have contributed to the development of the SIF formulas for DCDC. However, discrepancies exist between the derived formulas [9,10,11,12], hindering the application. Ref. [9] is one of the key papers in the DCDC literature, and it has been highly acclaimed. However, we found the results given in Ref. [9] were in question, because they were got from erroneous formulations based on models with simplified assumptions. Recently we published a two-page Brief Note in International Journal of Fracture [19] to alert the international fracture mechanics community to the errors in a short-crack model [9], but the underlying reasoning was omitted, the errors in a long-crack model [9] were not assessed, and especially, no revised formulas for DCDC were provided.

The present paper aims at developing revised SIF solutions for DCDC based on theoretical analysis and numerical computation. First, in Sections 2 and 3, the errors in Ref. [9], such as misuse of Saint-Venant's principle [20] and Bueckner's principle [21] in fracture mechanics, and erroneous formulations, are addressed and corrected respectively. Then, in Sections 4 and 5, some SIF formulas for DCDC in the literature are introduced, and particularly, our curve-fitted SIF formulas, which is the updated version of the previously derived formulas [18], are given. Further, in Section 6, the crack stability for the DCDC specimens is analyzed based on the variation trend of the dimensionless stress intensity factors given in a unified form, with exception for very small crack length, the crack is shown to grow stably in the whole fracturing process of DCDC even under a load-controlled condition. Conclusions and perspective are given in Section 7, followed by two appendixes elaborating the argument in Sections 2 and 3.

Note for comparison: Corrected equation for the counterpart in Ref. [9] is marked with a prime in the equation label.

2. SIF formulas for DCDC derived with two models of Ref. [9]

The short-crack model was based on a solution for a crack in an infinite plate, and the long-crack model was rooted in an Euler-Bernoulli beam solution.

2.1. Short-crack model [9]

First consider a Green's function for an infinite plate with a center crack of length $2a'$ subjected to two symmetrical pairs of concentrated tensile forces P applied on the crack surfaces, as shown in Fig. 2. Linear elastic fracture mechanics gives its SIF as [22]:

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Nomenclature

σ	compressive stress applied on DCDC
R	radius of circular hole
a	crack length
w	width
α	dimensionless crack length, $\alpha = a/R$
β	dimensionless width, $\beta = w/R$
t	thickness
M_0	bending moment
Y	correction factor, $Y = \sigma\sqrt{\pi R}/K_{\rm I}$
DCDC	double cleavage drilled compression

$$K_{\rm I} = \frac{P}{\sqrt{\pi a'}}\left(\sqrt{\frac{a'+x}{a'-x}} + \sqrt{\frac{a'-x}{a'+x}}\right) \tag{1}$$

where P is the force per unit thickness. Then referring to Fig. 1, modify it to be unbounded and uncracked, i.e. for an infinite plate with a center circular hole of radius R subjected to far field uniform compression stress of σ, according to the elasticity [20], the exact solution of the normal stress along the axial center line $y = 0$ and $|x| \geqq R$ is as follows:

$$\sigma_y(x) = \sigma\left(-\frac{R^2}{2x^2} + \frac{3R^4}{2x^4}\right) \tag{2}$$

This normal stress distribution at both hole sides, with some key turning points (extreme values, tension compression changeover) marked, is given by a figure in Appendix A.According to Bueckner's principle [21] and using Eqs. (1) and (2), for an infinite plate with a center crack, subjected to an opening stress $\sigma_y(x)$ in the regime of $R \leqslant |x| \leqslant R\sqrt{3}$ on the opposite crack surfaces, its SIF is

$$K_{\rm I} = \int_R^{R\sqrt{3}} \frac{\sigma_y}{\sqrt{\pi(a+R)}}\left(\sqrt{\frac{a+R+x}{a+R-x}} + \sqrt{\frac{a+R-x}{a+R+x}}\right)dx \tag{3}$$

where a is half the crack length, and R is the radius of the circular hole. Referring to Figs. 1, 2, and the figure in Appendix A, in order to get an explicit formula as replacement for the integration formula, the compression-induced distributed normal stress $\sigma_y(x)$ on the crack surfaces, is simplified into two pairs of concentrated forces, where P is the resultant force, $P = dR\sigma$, the distance between P and the center is eR, and d and e are constant numbers and determined by using the principle of static equivalence as below

$$dR\sigma = P = \int_R^{R\sqrt{3}} \sigma_y dx \;\Rightarrow\; d = \frac{\sqrt{3}}{9} \approx 0.19254 \tag{4}$$

$$PeR = \int_R^{R\sqrt{3}} x\sigma_y dx \;\Rightarrow\; e = \frac{3\sqrt{3}}{4}(2-\ln 3) \approx 1.1709 \tag{5}$$

If the width $2w$ of a DCDC is large, it becomes an infinite plate, and the SIF for the DCDC is given as

$$K_{\rm I} = \frac{d\sigma\sqrt{R}}{\sqrt{\pi(1+\alpha)}}\left(\sqrt{\frac{1+\alpha+e}{1+\alpha-e}} + \sqrt{\frac{1+\alpha-e}{1+\alpha+e}}\right) \tag{6}$$

where d and e are given in eq. (4) and eq. (5) respectively, and the dimensionless crack length is $\alpha = a/R$. Referring to Eq. (3), it was claimed in Ref. [9] that "The stress intensity factor due to this concentrated force is a very good approximation". However, it must be pointed out that Eq. (3) is itself in error, as demonstrated in the next Section 3.

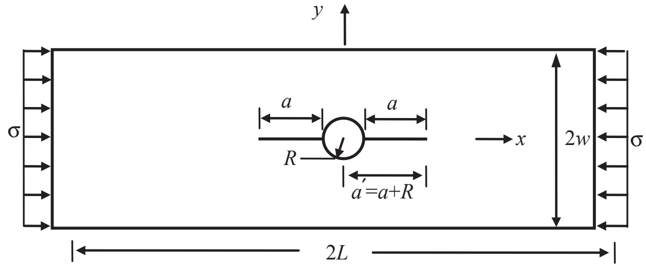

Fig. 1. Double cleavage drilled compression (DCDC) specimen.

Now consider the effect of finite width of DCDC. Let $\beta = w/R$, d in Eq. (6) is now a function of β, so $d(\beta)$represents the effect of the plate width. Clearly, referring to Eq. (4), when $\beta \to \infty$, $d = 0.19254$. In the considered range of β, i.e. $\beta \in \langle 2{,}5\rangle$, the following empirical formula for the width effect of the DCDC was obtained through linear fitting

$$\frac{d(\beta)}{d} = 5.7 - 0.75\beta \quad \beta \in \langle 2{,}5\rangle \tag{7}$$

This empirical formula is shown to be inaccurate [18].

Combining Eq. (6) and Eq. (7), the SIF formula of the DCDC based on the short-crack model is

$$K_{\rm I} = \frac{d\sigma\sqrt{R}(5.7-0.75\beta)}{\sqrt{\pi(1+\alpha)}}\left(\sqrt{\frac{1+\alpha+e}{1+\alpha-e}} + \sqrt{\frac{1+\alpha-e}{1+\alpha+e}}\right) \tag{8}$$

Eq. (8) is the SIF formula derived in Ref. [9] using the short-crack model.

2.2. Long-crack model [9]

Readers are advised to refer to Ref. [9] for the formulation, only a brief description is presented here. In order to distinguish, we use abbreviation Eq. to denote the equation given in the present paper, and eq. to denote the equation quoted from Ref. [9] and not shown here. Substituting eq. (25) into eq. (21) of Ref. [9], the SIF formula for DCDC with a long crack is expressed as

$$K_{\rm I} = \frac{2M_0}{tw^{3/2}} = \frac{2\sigma\sqrt{R}}{\sqrt{\beta}}\left(\frac{3+2\ln\beta}{4\beta} - \frac{1}{4\beta^3}\right) \tag{9}$$

where, t is the thickness, M_0 is the bending moment. Eq. (9) represents the same SIF formula as derived in Ref. [9]. Noticeably, the SIF is constant disregarding the extension of the crack, as the dimensionless crack length α does not appear in Eq. (9). The authors of Ref. [9,17] claimed that the crack growth is unstable when the crack enters the so-called "plateau regime". Further, based on the assumed distribution of the normal stress at the holed cross-section ($x = 0$, in Fig. 1), i.e. eq.

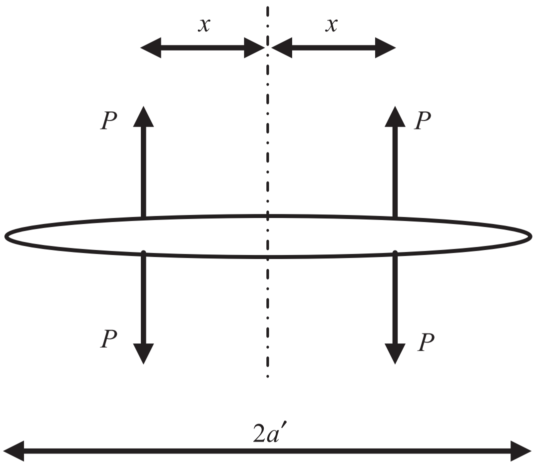

Fig. 2. Infinite plate with a center crack subjected to two symmetrical pairs of concentrated forces.