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Dynamic stress intensity factors for suddenly loaded structures using enriched finite elements

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ABSTRACT

In this paper, an efficient finite element formulation for stationary cracks subjected to dynamic impact loading is presented. For impact problems, where wave propagation effects dominate, the onset of rapid crack growth is strongly influenced by inertia effects, including stress wave reflections from geometric boundaries. Dynamic stress intensity factors generally attain maximum values that can be many times greater than their static counterparts. Because of this, there is a strong motivation for developing efficient computational techniques to evaluate this type of engineering problem in a relatively straightforward manner. In order to analyze the dynamic stress intensity factor problem efficiently, a computational technique that does not require a special crack tip mesh is of considerable value. The enriched finite element approach is shown to be a practical and effective technique for obtaining dynamic stress intensity factors, especially for cracks located on bimaterial interfaces, where there is inherent coupling between the mode I and mode II stress intensity factors. The enriched crack tip element approach utilizes the analytic asymptotic crack tip fields to directly compute the stress intensity factors. In this paper, fracture problems known to have two different types of crack tip singularities, subjected to elastic wave propagation effects during impact loading, are given as examples.

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1. Introduction

Reliability assessment of cracked structures subjected to sudden impact loading can involve considerable computational effort. The interaction of propagating elastic stress waves with the characteristic boundaries of a cracked structure invariably results in complex dynamic stresses that are greater in magnitude than what is observed in the corresponding static problem. The elevated strain rates during dynamic loading may result in brittle fracture behavior for a ductile material at stress levels significantly greater than the material's quasi-static yield stress, due to an increase in elastic properties and yield stress at high loading rates. The first "modern" scientific study of fracture under dynamic loading was published by Hopkinson [1] in 1872. Hopkinson measured the strength of iron wires subjected to a falling weight and found that failure of the wire depended on the velocity of impact and not the mass of the falling weight. He explained this surprising result in terms of interaction of the elastic waves that propagated axially along the wire. Following the development of the discipline of fracture mechanics, a number of studies related to dynamic fracture mechanics were published. Among the pioneering investigations, the work performed by Mott [2], Schardin et al., [3], Kerkhof [4,5], Yoffe [6] and Wells and Post [7] are notable. Since then, substantial progress in the field has been made and a vast number of publications have appeared in the literature. For example, studies of dynamic fracture mechanics problems given in the book by Sih [8], and review articles by Erdogan [9], Achenbach [10] and Freund [11] are highly referenced.

The finite element method has become the most popular technique for solving dynamic crack problems in recent years. This is because of the relative generality of the approach and the existence of a number of commercially available finite element programs that can be used to generate solutions for 3-D geometries. Unfortunately, finite element techniques will not yield accurate results if the stress singularity known to exist at the crack tip, is not properly taken into account. Stress intensity factors obtained from local stresses, or displacements, generally take advantage of crack tip elements that incorporate some form of the appropriate stress singularity at the crack tip. The enriched element method embeds the correct r and θ dependence of the stress singularity obtained from a known analytic solution for the asymptotic crack tip field [12]. The

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specific analytic form of the crack tip asymptotic field can depend on local details, such as material inhomogeneity and anisotropy. One practical advantage of the enriched crack tip element is that it does not require special meshing, or excessive mesh refinement, at the crack tip. Thus, automatic meshing can be taken advantage of to construct accurate finite element crack models. It should be noted, that in conjunction with the enriched crack tip element, transition elements should be included to maintain displacement compatibility between the enriched "crack-tip" elements and regular elements [12]. However, this is handled in a straightforward manner, since transition elements have the same geometric forms and nodal configurations as regular elements. In order to obtain accurate results using the enriched finite element method in dynamic fracture mechanics problems, mesh refinement and suitable time step intervals also need to be controlled.

In this paper, the objective is to review the fundamental dynamic solution methods for fracture analysis using enriched finite elements, followed by a few detailed numerical examples. The computational tool used in this study was a specialized fracture mechanics finite element code, FRAC3D, developed at Lehigh University for determining dynamic stress intensity factors. In addition to computing fundamental fracture parameters, the code provides nodal displacements and stresses at each time step of the dynamic solution process. The computed dynamic stress intensity factors are compared with static solutions and known classical elastodynamic examples from the literature. Finally, the utility of having specialized finite element software for the evaluation of dynamically loaded cracked structures is illustrated with a "T-shaped" interface crack problem, where no known previous solution exits. Of particular interest in this problem is the coupling that occurs between the mode I and mode II stress intensity factors; behavior that is explicitly contained in the enriched element formulation for an interface crack. Additional details related to the numerical formulation presented in this paper are given by Saribay in [13].

2. Dynamic finite element equations

The finite element equations that govern the dynamic response of a structure are derived by requiring a balance between the work of external forces and the work of internal, inertial, and viscous forces for any small kinematically admissible motion (i.e., any small motion that satisfies both compatibility and essential boundary conditions). For a single element, this work balance can be expressed as [14]:

$$\int_{Ve} \{\delta u\}^T \{F\}\, dV + \int_{Se} \{\delta u\}\{\phi\}^T\, dS + \sum_{i=1}^{n} \{\delta u\}_i^T \{p_i\} = \int_{Ve} \left(\{\delta\varepsilon\}^T \{\sigma\} + \{\delta u\}^T \rho \{\ddot{u}\} + \{\delta u\}^T \kappa_d \{\dot{u}\} \right) dV, \tag{1}$$

where $\{\delta u\}$ and $\{\varepsilon\}$ are respectively small virtual displacements and their corresponding strains, $\{F\}$ are body forces, $\{\phi\}$ are prescribed surface tractions (which typically are nonzero over only a portion of surface S_e, $\{p_i\}$ are concentrated loads that act at a total of n points on the element, $\{\delta u\}_i^T$ are the virtual displacements of points where loads p_i are applied, ρ is the mass density of the material, κ_d is a material-damping parameter (viscous damping), and volume integration is carried out over the element volume V_e.

Using the usual FEM notation, where $[N]$ represents the element interpolation function matrix and $\{d\}$ the unknown nodal displacement vector, we have for the displacement field $\{u\}$ (which is a function of both space and time) and its first two derivatives of time,

$$\{u\} = [N]\{d\}, \quad \{\dot{u}\} = [N]\{\dot{d}\}, \quad \{\ddot{u}\} = [N]\{\ddot{d}\}. \tag{2}$$

Combining (1) and (2) yields,

$$[M]\{\ddot{d}\} + [C]\{\dot{d}\} + \{r^{\text{int}}\} = \{r^{ext}\}, \tag{3}$$

where the element mass and damping matrices are defined as

$$[M] = \int_{Ve} \rho [N]^T [N]\, dV, \tag{4}$$

$$[C] = \int_{Ve} \kappa_d [N]^T [N]\, dV, \tag{5}$$

where ρ is the mass density and κ_d is the damping coefficient. The element internal force and external load vectors are defined as

$$\{r^{\text{int}}\} = \int_{Ve} \{B\}^T \{\sigma\}\, dV, \tag{6}$$

$$\{r^{ext}\} = \int_{Ve} [N]^T \{F\}\, dV + \int_{Se} [N]^T \{\phi\}\, dS + \sum_{i=1}^{n} \{p\}_i. \tag{7}$$

The internal force vector defined by (6) represents nodal forces at nodes caused by material straining. For linearly elastic material behavior, $\{\sigma\} = [E][B]\{d\}$ and (6) becomes

$$\{r^{\text{int}}\} = [k]\{d\}, \tag{8}$$

where $[k]$ is the element stiffness matrix and can be expressed as

$$[k] = \int_{Ve} [B]^T [E][B]\, dV. \tag{9}$$

In the preceding equations, $[B]$ is the strain-derivative matrix and $[E]$ is the elasticity matrix [14]. Combining Eqs. (3) and (8), we obtain the final system of equations that has to be integrated in time to obtain the unknown displacements $\{d\}$, velocities $\{\dot{d}\}$, and accelerations $\{\ddot{d}\}$,

$$[M]\{\ddot{d}\}_n + [C]\{\dot{d}\}_n + [k]\{d\}_n = \{r^{ext}\}_n, \tag{10}$$

where the subscript n denotes time $n \times \Delta t$, where Δt is the size of the time increment. More detailed information about the general dynamic finite element formulation can be found in [13,14].

3. Solution methods for dynamic problems

In dynamic simulations, two different types of direct integration techniques are usually exploited; explicit and implicit methods. The central difference method is an explicit method and uses the following equations for the velocity, $\{\dot{d}\}$ and acceleration vectors $\{\ddot{d}\}$:

$$\{\dot{d}\}^{n+1/2} = \{v\}^{n+1/2} = \frac{\{d\}^{n+1} - \{d\}^n}{t^{n+1} - t^n} = \frac{1}{\Delta t^{n+1/2}} (\{d\}^{n+1} - \{d\}^n), \tag{11}$$

$$\{\ddot{d}\}^n = \{a\}^n = \left(\frac{\{v\}^{n+1/2} - \{v\}^{n-1/2}}{t^{n+1/2} - t^{n-1/2}} \right). \tag{12}$$

As can be seen from (11) and (12), displacement vectors and velocity vectors are required at the midpoints of the time intervals, called half-steps or mid-point values. This type of formulation is particularly useful in problems where the time step increments are not constant. We can define the time increments for the general case by

$$\Delta t^{n+1/2} = t^{n+1} - t^n, \quad t^{n+1/2} = \frac{1}{2}(t^{n+1} + t^n), \quad \Delta t^n = t^{n+1/2} - t^{n-1/2}. \tag{13}$$

In the solutions provided in the following numerical examples section, all time steps were taken as constant. Eqs. (11) and (12) are

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