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Letter

Local buckling of thin plate on tensionless elastic foundations under interactive uniaxial compression and shear

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HIGHLIGHTS

- An analytical solution to the contact buckling behaviour of plates under combined compressive and in-plane shear loads is developed.
- Fitted formulas are derived for plates with clamped edges and simplified supported edges.
- Examples are given to demonstrate the practical application of the presented method.
- Finite element (FE) analysis is conducted to verify the analytical results.

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ABSTRACT

This paper uses a mathematical method to develop an analytical solution to the local buckling behaviour of long rectangular plates resting on tensionless elastic Winkler foundations and under combined uniform longitudinal uniaxial compressive and uniform in-plane shear loads. Fitted formulas are derived for plates with clamped edges and simplified supported edges. Two examples are given to demonstrate the application of the current method: one is a plate on tensionless spring foundations and the other is the contact between the steel sheet and elastic solid foundation. Finite element (FE) analysis is also conducted to validate the analytical results. Good agreement is obtained between the current method and FE analysis.

Plate elements are one of the main components in load-bearing structures and have wide engineering applications in the mechanical, marine, aeronautical, and civil industrial fields. Local buckling of plate elements often occurs when the structures are subjected to compressive and/or shear loadings. The buckling load is one of the governing criteria in steel and composite structural design.

For steel only structures, a plate element is able to buckle laterally both positively and negatively, which is called "bilateral buckling". The bilateral buckling phenomenon of an isotropic plate under combined in-plane shear and uniaxial compression is well understood. The interaction between ultimate compression and shear has been derived as [1, 2] $R_x + R_{xy}^2 = 1$, $R_x = \dfrac{\sigma_x}{\sigma_{xcr}}$, and $R_{xy} = \dfrac{\tau_{xy}}{\tau_{xycr}}$, where σ_x, τ_{xy}, σ_{xcr}, and τ_{xycr} are the maximum compressive stress during buckling, the maximum shear stress during buckling, the critical stress under pure compressive load alone, and the critical stress under pure uniform shear load alone, respectively.

However, for skin buckling behaviour in composite structures, the support from core materials has to be taken into account. An effective method is to simulate the core material as foundation using a rigid foundation model, either a one-parameter elastic model (like Winkler foundation [3]) or a two-parameter elastic model (like Pasternak foundation [4]). The skin buckling phenomenon is a kind of contact buckling problem, which has been extensively studied, especially for plates under pure compression or pure shearing loads, e.g. the local buckling analysis of plate under compression plate [5-9], local buckling analysis of plate under in-plane shear loads [10-12], and post-

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buckling analysis [13-15]. For thin-plates under combined loads, current research is focused on skins in unilateral contact with rigid materials. For example, local buckling behaviour of a finite plate with rigid medium under combined loads of bending, compression and shear [16-18] have been investigated. However, the previous combination buckling studies were focused on the rigid foundation, and the elastic tensionless foundation (between the rigid tensionless foundation and without foundation) was not taken into account to address the influence of elastic tensionless foundation on the contact buckling coefficient under combined shear and compression.

The current study addresses the foundation deformation effect on contact buckling behaviour under combined loads. An infinitely long, thin isotropic plate under interactive in-plane shear and uniaxial compressive loads, constrained on a tensionless Winkler foundation is investigated. Both clamped and simply supported edge conditions are considered. The fitted formula of the thin plate is obtained based on the analytical solution. In addition, the analytical results are compared with finite element (FE) simulation using ABAQUS.

A long plate under combined uniform in-plane shear and uniform longitudinal uniaxial compressive loads is illustrated in Fig. 1. The buckling half waves consist of inward half waves and outward half waves. If the foundation is considered, two kinds of zones occur between the plate and foundation, i.e. contact zones (inward half waves) and non-contact zones (outward half waves). For easy mathematical expression, two local Cartesian coordinate systems (x_1, y, w_1) and (x_2, y, w_2) were introduced in this study.

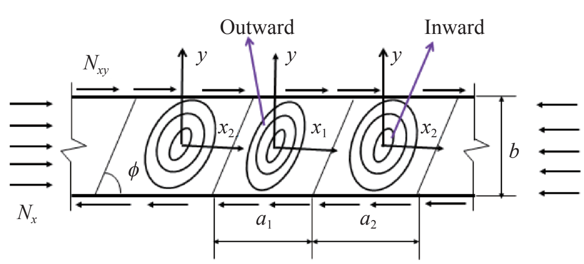

Fig. 1. Geometry of a long plate under combined loadings.

For the local buckling of infinite length thin plates constrained on an elastic foundation under combined uniform longitudinal uniaxial compressive and uniform in-plane shear loads, the governing equation may be written as follows

$$
\begin{aligned}
&Dw_{i,x_ix_ix_ix_i} + 2Dw_{i,x_ix_iyy} + Dw_{i,yyyy} + N_x w_{i,x_ix_i} \\
&\quad + 2N_{xy} w_{i,x_iy} = q_i, \quad |x_i| \le a_i/2,
\end{aligned} \tag{1}
$$

where D, N_x, and N_{xy} are the plate flexural stiffness, longitudinal uniaxial compressive, and shear force, respectively, which can be expressed as:

$$D = \frac{E^s t^3}{12\left[1-(\nu^s)^2\right]}, \tag{2a}$$

$$N_x = \sigma_x t, \tag{2b}$$

$$N_{xy} = \tau_{xy} t, \tag{2c}$$

where E^s, ν^s, t, σ_x, and τ_{xy} are Young's modulus, Poisson's ratio, plate thickness, longitudinal uniaxial compressive stress, and shear stress, respectively.

$$q_i = \begin{cases} 0, & |x_1| \le a_1/2 \text{ for non-contact zone}, \\ q_2(x_2, y), & |x_2| \le a_2/2 \text{ for contact zone}. \end{cases} \tag{3}$$

Assuming the following equations

$$K_x = \frac{b^2 t \sigma_x}{\pi^2 D}, \tag{4a}$$

$$K_{xy} = \frac{b^2 t \tau_{xy}}{\pi^2 D}, \tag{4b}$$

Eq. (1) may be rewritten as

$$
\begin{aligned}
&w_{i,x_ix_ix_ix_i} + 2w_{i,x_ix_iyy} + w_{i,yyyy} + K_x \frac{\pi^2}{b^2} w_{i,x_ix_i} \\
&\quad + 2K_{xy} \frac{\pi^2}{b^2} w_{i,x_iy} = q_i/D, \quad |x_i| \le a_i/2.
\end{aligned} \tag{5}
$$

Assuming $w_i(x_i, y) = f_i(\bar{x}_i) g(y)$ and the foundation is a Winkler foundation, Eq. (3) can be written as

$$q_i(x_i, y) = -k_i w_i(x_i, y) = -k_i f_i(\bar{x}_i) g(y), \tag{6}$$

where g and f_i (i = 1, 2) are the lateral direction (y-axis) and the longitudinal direction (x-axis) buckling mode function, respectively; k_i is the factor of stiffness and k is non-zero and zero for contact areas and for non-contact areas, respectively.

In Eq. (6), according to Fig. 1, $\bar{x}_i$ can be expressed as

$$\bar{x}_i = x_i - y\frac{1}{\tan\phi} = x_i - yc, \tag{7a}$$

$$c = \frac{1}{\tan\phi}, \tag{7b}$$

where ϕ is the skewed angle and shown in Fig. 1. Therefore, w_i may be presented as a function of c.

$$w_i(x_i, y) = f_i(\bar{x}_i) g(y) = f_i(x_i - yc) g(y), \tag{8a}$$

$$w_{i,x_ix_i} = f_i'' g, \tag{8b}$$

$$w_{i,x_ix_ix_ix_i} = f_i'''' g, \tag{8c}$$

$$w_{i,x_iy} = -cf_i'' g + f_i' g', \tag{8d}$$

$$w_{i,x_ix_iyy} = c^2 f_i'''' g - 2cf_i''' g' + f_i'' g'', \tag{8e}$$

$$w_{i,yyyy} = c^4 f_i'''' g - 4c^3 f_i''' g' + 6c^2 f_i'' g'' - 4cf_i' g''' + f_i g'''', \tag{8f}$$

where the superscript " ' " indicates the differentiation about x or y.

Substituting Eq. (8) into Eq. (5), Eq. (5) can be rewritten as

$$
\begin{aligned}
&\left(1+2c^2+c^4\right) f_i'''' g - \left(4c+4c^3\right) f_i''' g' + \left(2+6c^2\right) f_i'' g'' - \\
&\quad 4cf_i' g''' + f_i g'''' + K_x \frac{\pi^2}{b^2} f_i'' g + \\
&\quad 2K_{xy} \frac{\pi^2}{b^2}\left(-cf_i'' g + f_i' g'\right) + \frac{k_i f_i g}{D} = 0.
\end{aligned} \tag{9}
$$