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Compressed sensing based multi-rate sub-Nyquist sampling system

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Abstract

Signal sampling is a vital component in modern information technology. As the signal bandwidth becomes wider, the sampling rate of analog-to-digital conversion (ADC) based on Shannon-Nyquist theorem is more and more high and may be beyond its capacity. However the analog to information converter (AIC) based on compressed sensing (CS) is designed to sample the analog signals at a sub-Nyquist sampling rate. A new multi-rate sub-Nyquist sampling (MSS) system was proposed in this article, it has one mixer, one integrator and several parallel ADCs with different sampling rates. Simulation shows the signals can be reconstructed in high probability even though the sampling rate is much lower than the Nyquist sampling rate.

Keywords CS , sub-Nyquist rate, AIC, MSS

1 Introduction

With rapid development of wireless communication technology, the demand for information is increasing dramatically. The signal bandwidth becomes wider and wider to satisfy the increasing data volume. In traditional digital signal processing which inherently relies on sampling process, the ADC requires the sampling rate must be at least twice of the bandwidth according to the Shannon-Nyquist theorem to guarantee the reconstruction of the band-limited signal. The high frequency and high resolution ADC is one of the main performance limiters in advanced communication applications, where the bandwidth is high and the sampling rate is beyond the capacity of ADC. In the case of the research based on traditional ADC, the acquisition hardware, the subsequent storage and digital signal processors are facing with great challenges.

Periodic non-uniform sampling is a popular approach to reduce the sampling rate. Multicoset sampling is a specific strategy of this type [1]. Instead of implementing a single ADC at a high Nyquist sampling rate f_{NYQ}, interleaved ADCs use P devices at rate f_{NYQ}/P with appropriate time shifts. However, time interleaving has two fundamental limitations. First, the P low-rate samplers have to share an analog front-end which must tolerate the input bandwidth f_{NYQ}. With today's technology the possible front-ends are still far below the wideband regime. Second, maintaining accurate time shifts, on the order of $1/f_{\mathrm{NYQ}}$, is difficult to implement.

Fortunately, recent work in CS provided ways to sample sparse or compressible signals efficiently at a sub-Nyquist rate [2–3]. CS suggested that the signal characteristics could be fully captured by a number of projections which are fewer than those required by Nyquist theorem and reconstructed from them lossless. CS reveals a useful theorem that the sampling rate is determined based on the actual information contents rather than the signal bandwidth. This theorem has found a wide range of applications in communication, such as channel estimation, sensor network and cognitive radio.

Kinds of practical methods to implement the AIC for sampling at a sub-Nyquist rate were presented [4–8]. AIC usually consists of three main components: demodulation, integral and uniform sampling. [4] extended the exiting CS framework to analog signals and the AIC has only one path. Another practical sampling system which was inspired by Ref. [4] was presented in Ref. [5], it consists of a bank of

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random demodulators and ADCs running in parallel. The corresponding shows that this system has much lower sampling rate but consumes much resource if the architecture contains a large number of parallel paths.

In order to provide design flexibility and scalability, a parallel segmented compressed sensing (PSCS) structure was adopted [6–8]. The original signals are segmented and simultaneously transmitted to several parallel paths, then are fed into output after demodulation, piecewise integral and low speed sampling. PSCS reduces the number of paths by increasing the sampling rate. But it still has several parallel paths which are independent of each other. Each path has separated demodulation, piecewise integral and uniform sampling. PSCS structure consume much resources if a large number of parallel paths are contained.

A new MSS system was proposed in this article to make the AIC be easily implemented. Its architecture only has one multiplier and one integrator. The original signals are first demodulated and integrated during the whole signal period, and then are simultaneously sent to different ADCs to be sampled at different rates. Compared with PSCS structure, this architecture is simpler and consumes fewer resources.

The remainder of this article is organized as follows. In Sect. 2 we introduce the multicoset sampling and AIC background. Sect. 3 describes the MSS system and gives some analysis. Simulation results are shown in Sect. 4 and conclusions are made in Sect. 5.

2 Multicoset sampling and AIC background

2.1 Multicoset sampling

Multicoset sampling involves periodic nonuniform sampling of the Nyquist-rate sequence $x(nT_{\text{NYQ}})$, where $T_{\text{NYQ}} = 1/f_{\text{NYQ}}$. Let P be a positive integer, and $C = \{c_p\}_{p=1}^{P}$ be a set of p distinct integers with $0 \leqslant c_p \leqslant P-1$. Multicoset samples consist of p uniform sequences, called cosets, with the pth coset defined by

$$x_{c_p}[n] = x(nPT_{\text{NYQ}} + c_p T_{\text{NYQ}}); \quad n \in \mathbb{Z} \tag{1}$$

Only $p < P$ cosets are used, so that the average sampling rate is $p/(PT_{\text{NYQ}})$, lower than the Nyquist sampling rate.

A possible implementation of the multicoset sampling is depicted in Fig. 1. The building blocks are p uniform samplers at rate $1/(PT_{\text{NYQ}})$, where the ith sampler is shifted by $c_p T_{\text{NYQ}}$ from the origin. Although this scheme seems intuitive and straightforward, ADCs practically introduces an inherent bandwidth limitation, which distorts the samples. To avoid these distortions, an ADC with matching the Nyquist rate of the input signal must be used, even if the actual sampling rate is below the maximal conversion rate [1]. Thus, for wideband applications that cannot afford the size or expense of an optical system, the multicoset sampling becomes impractical. Another limitation of multicoset sampling, which also exists in the optical implementation, is maintaining accurate time delays between the ADCs of different cosets. Any uncertainty in these delays hobbles the recovery from the sampled sequences.

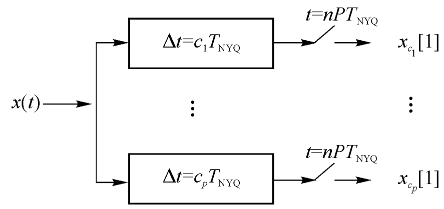

Fig. 1 Schematic implementation of multicoset sampling

2.2 AIC architecture design

2.2.1 CS theory

CS provides a framework for acquisition of a discrete-time signal which is sparse or compressible in somewhat sparsity basis. It is supposed that $x \in \mathbb{R}^N$ is an N-point real-valued discrete-time signal. Then $\boldsymbol{x}$ can be represented in an arbitrary basis $\{\psi_i\}_{i=1}^{N}$ for $\mathbb{R}^N$ with the weighting coefficients $\{\theta_i\}_{i=1}^{N}$. The signal $\boldsymbol{x}$ is represented as $\boldsymbol{x} = \sum_{i=1}^{N} \theta_i \psi_i = \boldsymbol{\Psi\Theta}$, where $\boldsymbol{\Psi}$ is an $N \times N$ matrix using ψ_i as columns, $\boldsymbol{\Theta}$ is the coefficient vector composed by the coefficients $\{\theta_i\}_{i=1}^{N}$. A signal is K-sparse in the basis $\boldsymbol{\Psi}$ if only K ($K \ll N$) significant elements are nonzero.

The useful information in the compressed signal can be captured by the non-adaptive linear projection. The random measurement for K-sparse signal $\boldsymbol{x}$ can be expressed as $\boldsymbol{y} = \boldsymbol{\Phi x} = \boldsymbol{\Phi\Psi\Theta}$, where $\boldsymbol{y}$ is an $M \times 1$ vector and $\boldsymbol{\Phi}$ is an $M \times N$ ($M \ll N$) matrix. In order to recover the original signal $\boldsymbol{x}$, the matrix $\boldsymbol{A} = \boldsymbol{\Phi\Psi}$ must satisfy the restricted isometry properties (RIP) [2]. That is, the measurement matrix $\boldsymbol{\Phi}$ must be incoherent with the