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The effective stress law for stress-sensitive transversely isotropic rocks

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ABSTRACT

This paper first presents a review of the development of the concept of effective stress, followed by major experimental and theoretical studies carried out to estimate the Biot's coefficient. It then uses the constitutive equations for vertically transverse isotropic (VTI) reservoirs, like coal, derived using the principles of thermodynamics for estimation of the Biot's coefficients in the vertical and horizontal directions. Laboratory data for tests conducted on two coal types retrieved from different geologic settings and geographical locations was used to carry out the modeling and validation exercise. Evidence is presented that values of Biot's coefficient can be greater than one, proposed by Biot to be the limiting value, for sorptive rocks. To address this, the term Biot's coefficient is replaced with "effective stress coefficient". Finally, this paper discusses the pressure- and stress-dependent behavior of the Biot's coefficient. The results clearly show that the estimated values of Biot's coefficients in both vertical and horizontal directions are different, varying with pressure for methane depletion but remaining constant for helium depletion. At the same time, the nature of Biot's coefficient, re-termed as effective stress coefficient, was found to be greater than unity for methane depletion. As a last step, a conceptual physical model is proposed to explain the pressure-dependent variation of effective stress/Biot's coefficients in terms of the contact area between grains. Based on the findings that the effective stress coefficient decreases with pressure, it is concluded that the effective vertical stress would increase significantly with depletion which, in turn, would result in shear failure and increased permeability.

1. Introduction

The concept of effective stress is central and critical to reservoir geomechanics when dealing with porous rocks with fluid partially or fully residing in it[1,2]. It is fundamental to several applications in reservoirs, like reservoir compaction[3–7], flow problems[8–12], PV (pressure-volume) problems, temperature changes[13–15] and other rock responses that are applicable to, and measured in, a reservoir (be it water, oil or natural gas). Reservoir compaction with depletion and the associated stress path depends on the effective stress[3,4,16–23] given that reservoir deformation is controlled by the effective stress, not total stress. It is well accepted that permeability variation in a reservoir is also controlled by changes in the effective stress[21,24–29]. The stress-based models for prediction of permeability variation with depletion in coalbed methane reservoirs use effective stress as an input parameter. In addition, the sensitivity of these models to variation in effective stress is significant. Hence, an improved understanding of the effective stress law would improve the performance of these models when evaluating the long-term production behavior of reservoirs. Various other processes and reservoir properties that depend on effective stress are the strength, acoustic velocity and capillary pressure[30].

The concept of effective stress, over ninety years old, was first defined as the difference between external stress and pore pressure[1]. The concept has since been revised several times and researchers continue to work on it, given its importance and criticality during modeling and simulation of oil/gas production in order to make reliable future projections. The laws of effective stress have evolved over the years from ones dealing with soil to isotropic rocks, followed by anisotropic rocks and, finally, for sorptive rocks, like coal. Dealing with the last two problems, anisotropy and sorptive rock, poses a major challenge that is somewhat recent and continues to be a topic of research worldwide.

The term, effective stress, was originally developed to deal with saturated and unsaturated rocks, just like for any non-porous material by substituting the total stress by effective stress. By definition, it is the stress acting on the matrix of the porous solid, estimated by cancelling the effect of pore pressure. A correct expression for effective stress can help in dealing with saturated media by replacing total stress in constitutive equations, failure criterion and flow problems with it. This paper first presents a brief review of the effective stress laws and their evolution over time. It then presents a means to estimate the effective stress for transversely isotropic and sorptive rocks.

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2. Overview

2.1. Fundamental studies

The first concept of effective stress was presented by Terzaghi[1] and was related primarily to soil mechanics. Terzaghi's definition of effective stress, however, was for one-dimensional stress only. In addition to being one dimensional, the definition was for constant load condition and saturated soils with incompressible grains. The definition of effective stress was simply the difference between applied stress and pore pressure, given as:

$$\sigma^{eff} = \sigma - p \tag{1}$$

where, σ_{eff} is the effective stress, σ is the total applied stress and p is the pore pressure. This definition is still widely used for rocks and soils as a good approximation. The first complete and rigorous attempt to define and explain the theory behind effective stress was made by Biot[2]. The assumptions associated with treatment of effective stress by Biot were isotropy of the material, reversibility of stress-strain, linear stress-strain behavior, small strains, incompressible water and unsaturated conditions, and applicability of Darcy's law. Biot's extension of the effective stress theory of Terzaghi was to three dimensions is given as:

$$\sigma_{ij}^{eff} = \sigma_{ij} - \alpha p \delta_{ij} \tag{2}$$

where, σ_{ij}^{eff} is the effective stress tensor, σ_{ij} is the total stress tensor, p is the pore pressure, δ_{ij} is the Kronecker delta symbol and α is the Biot coefficient. Initially, Biot studied soil consolidation and proposed the following equation to estimate the coefficient for soil consolidation:

$$a = \frac{E}{3 \quad (1 - 2\upsilon)H} \tag{3}$$

where E is the Young's modulus, υ is the Poisson's ratio and H is the effective modulus, defined for the porous media. Eq. (3) was a stepping stone in the formulation of the effective stress expression proposed by Biot[31].

Skempton[32] and Bishop[33] presented an expression for the Biot coefficient in terms of contact area between particles per unit gross area of the material. Geertsma[34] proposed an expression for the coefficient for Biot's effective stress law as:

$$\alpha = 1 - \frac{K}{K_s} \tag{4}$$

where K and K_s are the bulk and grain moduli of the rock respectively. A similar expression was proposed by Skempton[32], given as:

$$\alpha = 1 - \frac{C_s}{C} \tag{5}$$

where C_s and C are the grain and bulk compressibilities of the porous media. Suklje[35] modified the above expression with addition of the porosity term (ϕ) as:

$$\alpha = 1 - (1 - \phi)\frac{K}{K_s} \tag{6}$$

Nur and Byerlee[36] provided experimental validation for Eq. (4) and theoretical derivation for the expression of Biot coefficient. Using the results of a series of experiments conducted on weber sandstone, they concluded that using the expression of Biot[2] and Eq. (4) gives better result than that obtained using Terzaghi's expression.

2.2. Variation of Biot coefficient

Researchers worldwide have reported experimental work suggesting that Biot's coefficient varies with changes in confining stress and pore pressure. Fatt[37] measured values of the Biot's coefficient for Boise sandstone using kerosene as the pore fluid and reported that, with varying confining stress, the value of α varied between 0.77 and 1.0. Todd and Simmons[38] studied the effect of pore pressure and confining stress on seismic velocity and their derivation of α was later extended as[39–41]:

$$\alpha = 1 - \frac{\left.\frac{\partial Q}{\partial p}\right|_{\sigma}}{\left.\frac{\partial Q}{\partial \sigma}\right|_{p}} \tag{7}$$

where Q is any measured physical quantity of a core sample, like strain or wave velocity[42]. Christiansen and Wang[39] reported values of α ranging from 0.5 at high stresses to 0.89 at low stresses for Berea sandstone by measuring the dynamic properties and deformations. Warpinski and Teufel[30] reported the variation of α with changes in stress and pore pressure for tight sandstone and chalk from 0.65 to 0.95. Franquet and Abass[43] conducted experiments on sandstone to propose that the value of α decreases with increasing confining pressure. A recent study by Ma and Zoback[42] reported, based on experimental work on samples from Middle Bakken, that there is dependence of Biot's coefficient on confining stress and pore pressure. They concluded that the variation of α is significant at higher pore pressure and stress conditions, resulting in failure of Terzaghi's effective stress law. They used the method proposed by Christensen and Wang[39] for estimation of the Biot's coefficient using dynamic as well as static measurements, a modification of the method first proposed by Todd and Simmons[38]. However, they commented that using dynamic method is not suitable for estimation of the coefficient because both elastic and non-elastic strains occur in a reservoir and using a static test is, therefore, better when estimating the value of α.

2.3. Anisotropy of Biot's coefficient

Since most of the above experimental work and theoretical derivation dealt with isotropic rocks, such as, sandstone and limestone, the above form of effective stress law cannot be accepted in its current form for anisotropic rocks, like coal. These exhibit a special kind of anisotropy, termed vertical transverse isotropy (VTI), exhibiting isotropy in the two horizontal directions but vertical anisotropy. Hence, defining an effective stress law for these rock-types would be more appropriate for use in reservoir geomechanics problems in coalbed methane reservoirs. Carroll[44] developed equations for anisotropic effective stress law for elastic deformation. The derivation is simple and follows the same procedure as that followed by Nur and Byerlee[36]. Caroll[44] proposed that effective stress law requires two constants for transversely isotropic medium and three for an orthotropic medium. Considering both anisotropic pore geometry and intrinsic anisotropy, Caroll proposed that using six material constants, two of which are for solid material and four porous media constants, the two Biot's coefficients can be estimated for a transversely isotropic medium. Coussy[45] presented a set of poroelastic equations, derived thermodynamically and based on energy balance, for transversely isotopic media, as:

$$\begin{aligned}
\sigma_{11} &= C_{11}\varepsilon_{11} + C_{12}\varepsilon_{22} + C_{13}\varepsilon_{33} - b_1 N[(\phi_c - \phi_{c0}) - b_1(\varepsilon_{11} + \varepsilon_{22}) \\
&\quad - b_3\varepsilon_{33}] \\
\sigma_{22} &= C_{12}\varepsilon_{11} + C_{11}\varepsilon_{22} + C_{13}\varepsilon_{33} - b_1 N[(\phi_c - \phi_{c0}) - b_1(\varepsilon_{11} + \varepsilon_{22}) \\
&\quad - b_3\varepsilon_{33}] \\
\sigma_{33} &= C_{13}\varepsilon_{11} + C_{13}\varepsilon_{22} + C_{33}\varepsilon_{33} - b_3 N[(\phi_c - \phi_{c0}) - b_1(\varepsilon_{11} + \varepsilon_{12}) \\
&\quad - b_3\varepsilon_{33}] \\
\sigma_{23} &= 2C_{44}\varepsilon_{23} \\
\sigma_{31} &= 2C_{44}\varepsilon_{31} \\
\sigma_{12} &= 2\frac{(C_{11} - C_{12})}{2}\varepsilon_{12} \\
p_c &= N[(\phi_c - \phi_{c0}) - b_1(\varepsilon_{11} + \varepsilon_{22}) - b_3\varepsilon_{33}]
\end{aligned} \tag{8}$$

where,